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SIMPLIFIED 2-PGD BASED SMOOTHING OF S-CURVE DERIVED UH

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ABSTRACT

A simplified two-parameter gamma distribution (2-PGD) has been employed to obviate the erratic oscillations frequently observed in the recession part of conventional S-curve derived desired-duration (τ-hr) unit hydrograph (UH). The area under the derived 2-PGD based oscillation-free τ-hr UH is guaranteed to be unity as requisite for the UH derivation. It has identical runoff volume as of estimated equilibrium discharge (Q_{eq}) of S-curve which justifies the accuracy of the proposed approach. The 2-PGD proves its supremacy over conventional-cumbersome manual smoothing of UH by generating smooth shaped UH with reasonable manual efforts. It only needs the peak discharge (q_p) and time to peak (t_p) of UH as input in order to reproduce the same with ordinates spaced at any time step. Overall proposed procedure skips unjustifiable interpolation of parent UH and erroneous manual efforts made to achieve a smooth shaped τ-hr UH of unit volume.

Key words: *S-curve/S-hydrograph, Desired-duration UH/SUH, 2-PGD.*

INTRODUCTION

Abnormal shaped UHs resulted due to various factors such as non-uniform distribution and varying characteristics of rainfall, unsaturated watershed conditions etc. create the need of adjusting them to a desired common duration of effective-rainfall (Patil and Mishra, 2016). Under such situation conventional S-hydrograph technique is best suited as a mediator (Dooge, 1973). The S-hydrograph/S-curve is the direct runoff hydrograph due to continuous uniform effective-rainfall intensity of infinite duration (Chow et al., 1988; Singh, 1992), and is theoretically smooth.

The S-curve based τ-*hr* UH derivation is not as easy as it appears. The S-curve ordinates may only be known at a time step (Δt) identical as of parent UH duration (*D-hr*) it is derived from. If desired UH duration is not an integer multiple of parent UH duration, interpolation of parent UH is must and then only parent S-curve can be derived. It is generated by adding a series of unit-duration parent UHs lagged by unit duration successively. A τ-*hr* UH can be derived by displacing the parent S-curve by τ-*hr*, taking the difference of both the S-curves, and then normalizing the volume. After such interpolation, it must be insured that the derived τ-*hr* UH represents unit runoff. If the derived hydrograph is normalized to unit volume, the resulting *t-hr* UH corresponds to rain falling at a rate of l/*t* depth units per hour for *t-hrs*. It always involves an uncertainty that, a smooth finite-period parent UH will produce a smooth S-curve or UH.

The numerical procedure, non-linearity of the system, inaccurate parent UH duration assumed, non-uniform runoff generation and errors in the basic data may result in oscillating rising and equilibrium stages of S-curve (hunting-effect) rather than approaching a constant value (i.e. Q_{eq}) at a time = time base (t_b) of parent UH. It is possible that, the S-curve oscillations may also lead to grossly erroneous ordinates (–ve tail end) of the τ-*hr* UH (Subramanya, 2013). Such abnormal sequence of ordinates cannot be ignored or left unadjusted. Therefore, in practice before taking the S-curve difference the parent S-curve is smoothed graphically. Additionally, the derived τ-*hr* UH is also adjusted manually for smoothness and unit volume, which is laborious and time consuming. Hence, there is a great need of an alternative approach that can reduce the human efforts maintaining the requisite accuracy.

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CONVENTIONAL S-CURVE APPROACH

i. Derive *D-hr* parent S-curve by adding a series of *D-hr* parent UHs lagged by *D-hr* successively.
ii. Derive τ-*hr* UH by displacing the *D-hr* parent S-curve by τ-*hr* (may be an integer multiple or fraction), taking the difference of both the S-curves and dividing it by the ratio (τ/D) at each time step.

S-CURVE DERIVED UH SMOOTHING USING 2-PGD

Simplified 2-PGD is proposed to eliminate the shortcomings of conventional S-curve approach in order to expedite the τ-*hr* UH derivation. As an example, a synthetic UH (SUH) whose q_p and t_p were derived using watershed characteristics, has been used as a *D-hr* parent UH for its conversion to τ-*hr* UH as follows:

i. Estimate q_p (hr^{-1}) and t_p (*hr*) of *D-hr* parent SUH using CWC (1984) approach and derive 2-PGD based complete shape of it with its ordinates spaced at $\Delta t = \tau$-*hr*.
ii. Convert the 2-PGD based *D-hr* parent SUH to τ-*hr* SUH using conventional S-curve approach. The τ-*hr* SUH exhibit oscillations in its recession limb as expected and yield a higher peak Q_p (m^3/s) at shorter t_p as requisite.
iii. To avoid such oscillations, reproduce τ-*hr* SUH using 2-PGD with its q_p and t_p obtained from Step (ii).

Any natural unit-duration UH can also be used as a parent hydrograph instead of 2-PGD SUH but it results much frequent oscillations in altered duration UHs.

SIMPLIFIED 2-PGD

The similarity between 2-PGD and theoretical hydrograph properties (i.e. shape, unit runoff volume and positive ordinates) motivate us to use 2-PGD as an analytical expedience to resemble hydrographs and hence applied in the present study to correct S-curve derived SUHs. Based on the concept of *n*-linear reservoirs of equal storage coefficient *K*, Nash (1959) and Dooge (1959) derived IUH in terms of gamma function as:

$$q = \frac{1}{K\Gamma(n)}\left(\frac{t}{K}\right)^{n-1} e^{\frac{-t}{K}}, K > 0, t > 0 \qquad (1)$$

Here, *n* and *K* defines shape of the IUH which are related as (Chow 1964):

$$K = \frac{t_p}{(n-1)} \qquad (2)$$

Using numerical simulation (i.e. Stirling formula; Abramowitz and Stegun, 1964) and optimization (Marquardt, 1962) Bhunya et al. (2003) derived approximate relations to estimate *n* for the known non-dimensional parameter β which is a product of q_p and t_p.

$$n = 5.53\beta^{1.75} + 1.04 \qquad for \ (0.01 < \beta < 0.35) \qquad (3a)$$

$$n = 6.29\beta^{1.998} + 1.157 \qquad for \ (\beta \geq 0.35) \qquad (3b)$$

It is one of the most popular and widely accepted approaches to approximating the real system (Singh et al., 2014).

STUDY AREA AND DATA USED

The sub-watersheds defined by Railway bridge No. 50 (Saharanpur-Moradabad, N.R.) and 1232 (Bhojpur-Lalkua, N.E.R.) located in Upper Indo-Ganga Plains Subzone-1(*e*), were used to exemplify the proposed approach. These watersheds are termed as Bridge No. 1 and 2, here onwards. Physiographic characteristics (Table 1) of these watersheds have been obtained from flood estimation report of Upper Indo-Ganga Plains Subzone-1(*e*) developed by CWC and RDSO, Ministry of Railways (CWC, 1984).

Table 1: Physiographic characteristic of the selected sub-watersheds (CWC, 1984)

Physiographic Characteristics	Bridge No. 1	Bridge No. 2
Catchment area (*A*) in km^2	25.26	49.47
Length of longest stream (*L*) in *km*	15.00	16.19
Equivalent stream slope (*S*) in *m/km*	2.00	2.41

Table 2: Computation of 2-*hr* SUH parameters using CWC (1984) approach

Synthetic relationship used	Bridge No. 1	Bridge No. 2	Unit	
Peak discharge per unit area $q_{pc} = \frac{2.030}{(L/\sqrt{S})^{0.649}}$	0.438	0.443	$m^3/s\,/\,km^2$	(4)
Peak discharge $Q_P = q_{pc} \times A$	11.07	21.93	m^3/s	(5)
	$q_p = 0.158$	$q_p = 0.160$	hr^{-1}	
Lag time $t_l = \frac{1.858}{(q_{pc})^{1.038}}$	4.37	4.32	*hr*	(6)
Time to peak $t_p = t_l + \left(\frac{t_r}{2}\right)$	5.37	5.32	*hr*	(7)
Time base $t_b = 7.744 \times t_l^{0.779}$	24.44 ≈ 25	24.23 ≈ 25	*hr*	(8)
Volumetric equality condition Volume of SUH = $Q_{eq} = \frac{Ad}{0.36D}$	35.083	68.708	m^3/s	(9)

Volume of SUH= $\Sigma Q_i \times (\Delta t/D)$ when $\Delta t \neq D$-*hr* and Volume of SUH = ΣQ_i when $\Delta t = D$-*hr*,

Δt = time step, ΣQ_i = sum of all SUH ordinates, *d* = direct runoff depth.

ANALYSIS AND DISCUSSION OF RESULTS

Employing the physiographic characteristics of 23 sub-catchments of the Upper Indo-Ganga Plains Subzone-1(*e*), CWC (1984) proposed an approach (Eqs. 4-9, Table 2) for derivation of 2-*hr* SUH parameters, which is recommended by the Bureau of Indian Standards (IS 12094:2000). Applying the CWC (1984) approach over the physiographic characteristics (Table 1) of selected sub-watersheds 2-*hr* parent SUHs q_p (hr^{-1}) and t_p (*hr*) were estimated. The derived 2-PGD based 2-*hr* parent SUH is converted to 1-*hr* SUH using conventional S-curve approach and is further compared with 2-PGD based 1-

hr SUH to exemplify the utility of the 2-PGD in eliminating oscillations. To this end, the application to Bridge No. 1 is described.

For known *A*, *L* and *S*, the Q_P (= 11.07 m^3/s) is estimated using Eqs. (4-5), which is further converted to q_p (= 0.158 hr^{-1}) as required for 2-PGD SUH fitting. The t_p is estimated as 5.37 *hr* from Eq. (7). β (= $q_p\, t_p$) is estimated as 0.85. Since $\beta \geq$ 0.35, *n* is obtained as 5.68 from Eq. (3b). For known t_p and *n*, *K* is derived as 1.15 from Eq. (2). Thus, with known *n* and *K* the 2-PGD 2-*hr* parent SUH (Eq. 1, Fig. 1a, Table 3) is estimated. Interpolation has been skipped by 2-PGD which produces SUH ordinates at Δt =1-*hr*, and hence $\Delta t \neq D$-*hr*. The 2-PGD 2-*hr* SUH satisfies Eq. (9) with its estimated runoff volume $\Sigma Q_i \times (\Delta t / D) = \Sigma Q_i \times (1/2)$ = 35.083 m^3/s (Table 3). The tail end of the 2-PGD 2-*hr* SUH has been extended up to a time = t_b = 24.44 ≈ 25 *hr* (Eq. 8, Fig. 1a).

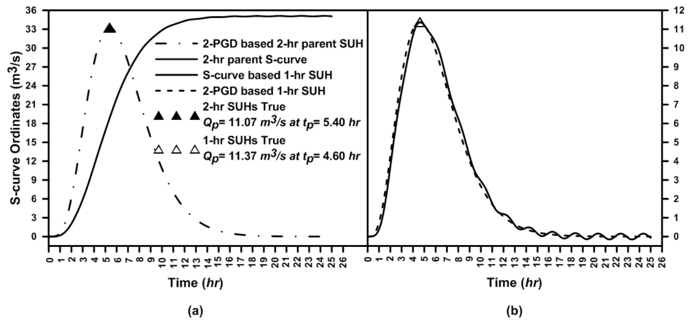

Fig 1: S-curve's and parent as well as desired-duration SUHs derived for Bridge No. 1

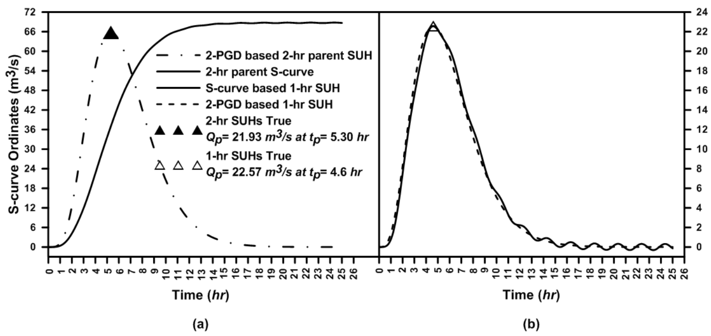

Fig 2: S-curve's and parent as well as desired-duration SUHs derived for Bridge No. 2

Table 3: S-curves and parent as well as desired-duration SUHs ordinates for Bridge No. 1 and 2

Bridge No.	**1**				**2**			
Time	**2-PGD based 2-*hr* SUH**	**2-*hr* parent S-curve**	**S-curve based 1-*hr* SUH**	**2-PGD based 1-*hr* SUH**	**2-PGD based 2-*hr* SUH**	**2-*hr* parent S-curve**	**S-curve based 1-*hr* SUH**	**2-PGD based 1-*hr* SUH**
hr	m^3/s	m^3/s	m^3/s	m^3/s	m^3/s	m^3/s	m^3/s	m^3/s
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.19	0.19	0.38	0.75	0.39	0.39	0.77	1.39
2	2.05	2.05	3.72	4.28	4.14	4.14	7.52	8.27
3	5.72	5.91	7.73	8.50	11.52	11.91	15.52	16.74
4	9.20	11.25	10.68	10.99	18.41	22.56	21.30	21.79
5	10.94	16.86	11.21	11.22	21.73	33.64	22.16	22.27
6	10.75	22.00	10.29	9.87	21.17	43.73	20.19	19.52
7	9.26	26.11	8.22	7.84	18.07	51.71	15.96	15.40
8	7.24	29.24	6.25	5.77	14.00	57.73	12.05	11.25
9	5.25	31.37	4.26	4.01	10.08	61.79	8.11	7.74
10	3.60	32.84	2.94	2.66	6.84	64.57	5.57	5.09
11	2.35	33.72	1.76	1.70	4.43	66.21	3.28	3.22
12	1.48	34.32	1.20	1.06	2.76	67.33	2.23	1.97
13	0.90	34.62	0.60	0.64	1.66	67.88	1.09	1.18
14	0.53	34.85	0.46	0.38	0.97	68.30	0.86	0.69
15	0.31	34.93	0.15	0.22	0.56	68.43	0.26	0.40
16	0.17	35.03	0.19	0.13	0.31	68.61	0.37	0.22
17	0.10	35.03	0.00	0.07	0.17	68.60	-0.02	0.12
18	0.05	35.08	0.11	0.04	0.09	68.71	0.21	0.07
19	0.03	35.05	-0.05	0.02	0.05	68.65	-0.11	0.04

20	0.02	35.09	0.08	0.01	0.03	68.73	0.16	0.02
21	0.01	35.06	-0.06	0.01	0.01	68.67	-0.13	0.01
22	0.00	35.10	0.07	0.00	0.01	68.74	0.15	0.01
23	0.00	35.06	-0.07	0.00	0.00	68.67	-0.14	0.00
24	0.00	35.10	0.07	0.00	0.00	68.74	0.14	0.00
25	0.00	35.07	-0.07	0.00	0.00	68.67	-0.14	0.00
Volume of SUH	35.083		35.065	35.087	68.707		68.672	68.715

To convert 2-PGD 2-*hr* SUH to 1-*hr* SUH conventionally, 2-*hr* parent S-curve (Fig. 1a & Table 3) has been derived by adding a series of 2-*hr* SUHs lagged by 2-*hr* successively. The resultant S-curve exhibit hunting-effect as expected and doesn't attain Q_{eq} at the t_b of 2-PGD 2-*hr* SUH. The derived S-curve is lagged by 1-*hr* and the difference of the two S-curves is divided by ratio τ/D=½ to achieve S-curve based 1-*hr* SUH (Fig. 1b & Table 3). Note, derived 1-*hr* SUH has higher Q_P (= 11.37 m^3/s) and shorter t_p (= 4.60 *hr*) than the 2-PGD 2-*hr* SUHs Q_P =11.07 m^3/s and t_p = 5.40 *hr* (Fig. 1a & 1b), but less correlated to Eq. (9) in terms of estimated runoff volume $\Sigma Q_i \times (1/2)$ = 35.065 m^3/s (Table 3). Again, as expected the oscillations of 2-*hr* parent S-curve are circulated to 1-*hr* SUH at its tail end (Fig. 1b).

The S-curves as well as SUHs ordinates derived are spaced at Δt = 1-*hr*. Hence, SUHs peaks occurred at the t_p which is not a whole number (e.g. 4.60, 5.40-*hr*) becomes invisible even if they exist. Such peaks are represented as 'true' peaks using different legends in Figs. (1 & 2) replicate the actual picture.

The oscillation-free 1-*hr* SUH (Fig. 1b & Table 3) has been reproduced by 2-PGD (Eq. 1) using S-curve based 1-*hr* SUHs $Q_P = 11.37\ m^3/s = q_p = 0.162\ hr^{-1}$ and t_p = 4.60 *hr* as input. The Fig. (1b) indicates reliable replacement of oscillatory S-curve 1-*hr* SUH by smooth 2-PGD 1-*hr* SUH having area under the curve as unity, as justified by the estimated runoff volume $\Sigma Q_i \times (1/2)$ = 35.087 m^3/s (Table 3) less deviated from the outcome of Eq. (9). By rearranging Eq. (9): $d=[35.087 m^3/s \times 0.36 \times D=2\text{-}hr] / [A=25.26 km^2]=1.00\ cm$. The 2-PGD SUHs also satisfies the condition that '*Time base* $_{shorter\ duration\ UH}$ < *Time base* $_{longer\ duration\ UH}$' as represented by the recession limb of 2-PGD 1-*hr* SUH that ceases (tends to zero) earlier than the 2-PGD 2-*hr* SUH (Figs. 1a & 1b, Table 3). Application of 2-PGD over Bridge No. 2 yields identical results as of Bridge No. 1 as represented by Figs. (2a & 2b) and Table (3).

CONCLUSIONS

The following conclusions can be drawn from the study:

i. The conventional-cumbersome approach of manually fitting a SUH through the derived point's viz. Q_p, t_p, W_{50}, W_{75}, WR_{50} and WR_{75} is obviated by 2-PGD SUH dependent only on q_p and t_p.

ii. Simplified 2-PGD automatically take care of the volume and non-negativity constraints in order to reproduce the desired-duration SUHs of exact peak and time to peak avoiding tail end oscillations observed conventionally.

iii. Simplified 2-PGD outperforms the conventional S-curve approach by optimum utilization of manual contribution and skipping the unproductive efforts and time involved.

iv. The analogousness, practical convenience and sound theoretical basis of the simplified 2-PGD approach assure its continued application in hydrograph studies.

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