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METEOROLOGICAL DROUGHT ANALYSIS USING COPULA FUNCTION FOR MARATHWADA REGION, MAHARASHTRA STATE, INDIA

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ABSTRACT

Drought is an extreme climatic event, which is recurring in nature. Drought occurs when the amount of precipitation is less than average precipitation for the region. Drought analysis is important due to its direct impact on water supply, agriculture and industry along with social and economic impact.Duration and severity are major characteristics of drought. In this study, the bivariate joint distribution of drought duration and severity is defined using bivariate copula function. 6-month Standardized Precipitation Index (SPI) is used to calculate drought duration and severity for average precipitation of the whole study area. The logistic and Birnbaum-saunders distributions are best fitted distribution for univariate frequency analysis of drought duration and severity. Student 't', Galambos, Clayton, Gumbel, Frank, AMH copula functions are used to find joint multivariate distribution. AIC, RMSE and NSE values are used to select the best fit copula function. Considering AIC, RMSE, NSE values and the dependence structure i.e closeness of isolines, the Galambos copula is found to be the best fit for time series data. Drought analysis is carried out using joint probability distribution between drought duration and severity. Drought risk is estimated based on a joint return period which gives important information for water resource planning and management.

Keywords: *Drought, SPI, Bivariate, Copula, Joint Return Period.*

INTRODUCTION

Drought is generally referred as lack of precipitation which is occurring frequently. When an amount of precipitation is below average precipitation value of a region, it is referred as a drought. Drought characteristics vary spatially from region to region. The development of drought is slow and it is hard to find the start and end period. The regional extent of drought is much larger as compared to other natural calamities such as flash floods and cloud burst. Drought event often affects water supplies by the deficit in soil moisture, depletion in stream flows and lowering of reservoir and groundwater level. Nature of drought is generally local which varies in space as well as on time scales. Therefore, the development of good management and mitigation system for drought events is necessary. In terms of definition and causes, droughtisa complex phenomenon. The analysis of drought event is difficult due to the fact that periodicity of droughts often varies from months to multi-years. Using univariate frequency analysis drought properties were investigated traditionally. Less than the actual calculation of related risk for water resource management happens due to univariate frequency analysis of drought events(Gonzalez and Valdes,2003). Chen, Singh, Guo, Mishra, & Guo (2013) highlighted that independent analysis that is univariate analysis of drought variables cannot give correlation between them whereas joint distribution gives a better description of drought characteristics.

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Traditional multivariate distribution is having various shortcomings; for example, marginal distributions are the same and different marginal distributions can't be used. Traditional multivariate methods use Pearson's linear correlation coefficient for finding dependence which is not an efficient measure of correlation when variables show non-linear dependence trend (Ganguli,2013).

Dependence between variables is studied using nonparametric measures such as Kendall's tau and Spearman's rho (Nelson, 2006).Copula function is applied to study dependence between drought variables i.e drought duration and severity. Drought variables are defined using the Standardized Precipitation Index. Exponential and gamma distributions are the most commonly used distribution functions fitted to duration and severity respectively for univariate frequency analysis. Probability values such as joint probability, conditional probability are found and isolines are plotted on observed data using best fitted copula function. Return period analysis is done using joint return period plots (Shiau, 2006; Serinaldi, Bonaccorso, Cancelliere, Grinaldi,2008; Shiau and Modarres, 2009;Mirabbasi, Fakheri, & Dinpashoh, 2012).Reddy and Ganguli (2013) calculated the conditional return period and plotted curves of Intensity-Area-Frequency. Sen's slope estimators and Man-Kendall non-parametric tests are used to find a spatial and temporal variation of drought episodes. Copula function's parameters are calculated using maximum-pseudo likelihood method. The goodness of fit test is performed to check the fitting of copula functions to data. For characterizing extreme data or events upper tail dependence test is conducted in which tail dependent coefficient is calculated to measure concurrence between extreme values**.** Ganguli (2013) evaluated Non-parametric UTDC (upper tail dependent coefficient)to overcome errors in parametric UTDC. Chen etal. (2013) used Metaelliptical and Archimedean copulasto developthe four-dimensional joint distribution of drought variables.

Markov model is applied to generate precipitation occurrence.

Sadegh, Ragno, & Agha Kouchak, (2017) developed multivariate copula analysis toolbox (MvCAT) which estimates copula parameters using Markov Chain Monte Carlo (MCMC) method along with local optimization and performs goodness of fit test for selecting best fit copula function. 26 copula functions are tested to select best fit copula function by using AIC, BIC, RMSE and NSEcriterion.

In this study, bivariate dependence analysis of drought events is studied for Marathwada region, Maharashtra state, India. Specific objectives of this paper are: i) To identify drought events along with its duration and severity based on 6-month SPI values.ii) To study dependence between drought duration and severity using best fitted copula function.

METHODOLOGY

Drought Characterization using SPI (Standardized Precipitation Index)

6-month standardized precipitation index is used for identifying drought events. SPI can be calculated for different timescales. SPI is generally probabilistic in nature i.e. probability distribution is fitted with rainfall record of the region and then transformed to the normal distribution, due to this normalization, SPI values above "0" shows wet conditions and below "0" shows dry conditions, where "0" correspond to long term average rainfall of the region. Based on the literature survey all SPI values below "-1" are considered for drought characterization. SPI is used because of its ease inhandling andflexibility of calculation at different timescales.

Following are the detail steps to calculate Standardized Precipitation Index for i^{th} year, j^{th} month and for a time scale of k months,

(i) Calculation of 'k' month moving average of rainfall time series X_{ij}^{k} (I = 1… n) for month j of year i, where every 'X' term is an addition of rainfall of previous k months.

(ii) Generally, Gamma distribution is fitted for average monthly rainfall data series varying from 3-month, 6-month till 24 months, which is given as follows:

$$g_X(x) = \frac{1}{\beta^{\xi}\Gamma(\xi)} x^{\xi-1} e^{\frac{-x}{\beta}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

Where,shape parameter is ξ, the scale parameter is β, and the gamma function at ξ is $\Gamma(\xi)$. Integrate equation (1) for getting cumulative probability distribution which is given as follows,

$$G_x(x) = \longrightarrow \int_0^x g_x(x)dx = \frac{1}{\beta^{\xi}\Gamma(\xi)}\int_0^x x^{\xi-1} e^{\frac{-x}{\beta}} dx \quad \ldots\ldots 2$$

(iii) Two parameter gamma distribution function doesn't consider '0' values but rainfall distribution contains '0' values, therefore, mixed distribution function which will consider '0' values are taken and CDF (cumulative distribution function) is defined, which is as follows,

$$F_X(X) = q + (1-q)G_x(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

Where, G(x): CDF for non zero rainfall, q: Probability of zero rainfall.

(iv) SPI (Standardized Precipitation Index) is given by following expression,

$$SPI = \psi^{-1}(F(x)) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4$$

Where, $\psi(\bullet)$: CDF of standard normal distribution, $\psi^{-}(\bullet)$: inverse of standard normal CDF.

SPI calculation requires only precipitation as an input variable. SPI has an intensity scale in which both positive and negative values are calculated, which correlates directly to wet and dry events.

Definition of Drought Properties

Drought properties are defined and calculated for studying the nature of association and dependence structure between them by using statistical methods. There are two main properties of drought event which are drought duration and drought severity. These properties are discussed below:

Drought Duration (D_d): Duration or time period for which SPI value remains below the threshold of '-1', this consecutive time interval are taken as drought duration or also called as drought length.

Drought Severity (S_D):Drought severity is defined as the summation of SPI values below "-1" within the period of drought. Drought severity can also be stated as the summation of negative SPI values below '-1'. The severity of a drought event in the i^{th}month is calculated by:

$$S_i = -\sum_{t=1}^{D} SPI_{i,t}$$

Depending upon values of SPI, drought event is characterized or divided into four groups from sr. no. 4 to 7 as shown in table 1.

Table 1: Classification of Drought Event-Based Upon SPI Values

Sr. no.	SPI	Class
1	+2 and above	Extremely wet
2	+1.5 to +1.99	Severely wet
3	+1 to +1.49	Moderately wet
4	-0.99 to 0.99	Near normal
5	-1 to -1.49	Moderately dry
6	-1.5 to -1.99	Severely dry
7	-2.0 and less	Extremely dry

Univariate Drought Duration and Severity Distribution

Univariate frequency analysis is generally used for an extreme climatic event, for finding the probability of occurrence of that event. Univariate probability distribution fitting is nothing but the fitting of distribution function to data points of single variable. Univariate fitting is done to do a prediction of the probability of variable of any particular event or to predict the frequency of occurrence of that variable. For this study best-fitted distribution for drought duration is logistic distribution and for drought severity is Birnbaum-Saunders distribution. Logistic distribution is given by the following equation,

$$\frac{\left(1+k\frac{x-\mu}{\sigma}\right)^{-1-1/k}}{\sigma\left(1+\left(1+k\frac{x-\mu}{\sigma}\right)^{-1/k}\right)^{2}}$$

Where,

K = shape parameter ≠ 0, σ = scale parameter > 0 and μ = location parameter

Birnbaum-Saunders distribution is given by the following equation,

$$f(x)=\left[\left(\frac{\beta}{x}\right)^{1/2}+\left(\frac{\beta}{x}\right)^{3/2}\right]exp\left[-\frac{1}{2\alpha^{2}}\left(\frac{x}{\beta}+\frac{\beta}{x}-2\right)\right]$$

Where, α = shape parameter > 0 and β = scale parameter > 0

Traditionally for probabilistic assessment of drought risk, univariate frequency analysis was employed. If hydro-climatological event is characterized by dependence among random variables, then the full estimation of the probability of an extreme event to occur cannot be estimated by univariate analysis and this may result in over and underestimation in the risk of drought. Therefore, instead of using the univariate frequency analysis, it is preferred to use multivariate probability distribution. Before fitting bivariate distribution to drought duration and severity using copula function, first, univariate distribution is fitted to drought duration and severity to get the marginal probability distribution values.

Bivariate Analysis of Drought Properties using Copula Function

Traditionally for bivariate dependence analysis, Pearson's linear correlation coefficient is used which is not an efficient measure of correlation when variables show non-linear dependence trend. Hence, it is intended to evaluate the marginal distribution of drought variables separately and find out a function that can join these marginal distributions while preserving the dependence structure of drought variables.In this context, by employing copulas drawbacks of conventional multivariate characterization can be solved. Copulas are joint distribution functions, used for finding the dependence between two or more random variables, obtained by combining their marginal distribution function. Two drought variables i.e drought duration and severity are defined and calculated and bivariate frequency analysis is studied.

Definition of Copula

Copula is a function that combines more than one distribution function with its constant one-dimensional margins(Nelsen, 2006).The backbone of copula theory is Sklar's theorem which says that there is a copula function C(u,v) for a joint distribution function H $_{X,Y}$ (x,y) for x,y. where,

$$H_{X,Y}(x,y)=C[F_X(x),F_Y(y)]$$

where, $F_X(x)$, $F_Y(y)$: Cumulative distribution functions of variables x and y.Conceptually copula function can be defined as, joint cumulative distribution function $H_{X,Y}(x,y)$ that joins individual CDF $F_X(x)$ and $F_Y(y)$ for variable x and y. Every pair of x and y refers to a point, $(F_X(x), F_Y(y))$, which lies within a square of size 1*1 and this ordered pair leads to a point, $H_{X,Y}(x,y)$ on the scale of 0 to 1 (Nelsen, 2006).

The uniqueness of copula function is that, the joint distribution function of two variables is not affected by their marginal distributions, hence any number of variables with different marginal distributions can be couple by using copula function. In this study copula functions used from Archimedean copula family are Ali-Mikhail-Haq copula, Frank copula, Clayton copula, and Gumbel - Hougaard copula.Galambos copula functionused in this study belongs to the extreme value class of copula family.Students "t" copula used in this study belongs to Meta elliptical class of copula family. Table 2 shows equations for different copula functions and domain for copula parameterθ.

Copula Parameter Estimation

The commonly used methods for estimation of copula parameters are (1) Method ofMoments(MOM) based on rank-based nonparametric measures of dependence which are Spearman's rho andKendall's tau (2) Exact Maximum Likelihood method (EML) (3) Inference from Margins (4) Maximum pseudo-likelihood estimation method or Canonical Maximum Likelihood. For calculating copula parameters and associated risk, MvCAT (Multivariate Copula Analysis Toolbox) uses a Bayesian framework with a residual-based Gaussian likelihood function. MvCAT is developed by Sadegh et al. (2017).

Goodness of Fit Test for Copula Function

Table 2: Important Types of Copula Functions

Type of copula	Equation of copula function	$\theta \in$
Ali-Mikhail-Haq	$\frac{uv}{1-\theta(1-u)(1-v)}$	[1, -1)
Frank	$-\frac{1}{\theta}\ln\left(1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right)$	$(-\infty,\infty)\backslash\{0\}$
Clayton	$\left[\max\left(u^{-\theta}+v^{-\theta}-1,0\right)\right]^{-1/\theta}$	$[-1,\infty)\backslash\{0\}$
Gumbel-Hougaard	$\exp\left(-\left[(-\ln u)^{\theta}+(-\ln v)^{\theta}\right]^{1/\theta}\right)$	$[1,\infty)$
Galambos	$C(u,v)=uv\exp\{[(-\ln u)^{-\theta}+\left(-\ln v\right)^{-\theta}]^{-\frac{1}{\theta}}$	$\theta \geq 0$
Students t	$\int_{-\infty}^{t_{\theta_2}^{-1}(u)}\int_{-\infty}^{t_{\theta_2}^{-1}(v)}\frac{\Gamma((\theta_2+2)/2)}{\Gamma(\theta_2/2)\pi\theta_2\sqrt{1-\theta_1^2}}\left(1+\frac{x^2-2\theta_1 xy+y^2}{\theta_2}\right)^{(\theta_2+2)/2} dxdy^c$	$\theta_1 \in [-1,1]$ and $\theta_2 \in (0,\infty)$

Various goodness of fit measures are taken into account to evaluate best fit copula function. AIC (Akaike Information Criterion) is used to estimate relative quality of statistical model for given set of data. AIC provides quality of each model with respect to another model. Copula functions are ranked according to their AIC values. Copula model with lowest AIC valueis given the highest rank. NSE (Nash-Sutcliffe Efficiency) and RMSE (Root Mean Square Error) are also two widely used measures of goodness of fit, which only focuses on minimization of residuals. A model fitting criterion according to RMSE and NSE is, RMSE value should be close to “0” and NSE value should be close to “1”.

Dependence Analysis using Copula Function

After selecting the best fit copula function by goodness of fit statistics, dependence analysis is studied for different copula functions. Dependence analysis using copula function is divided into two sections as copula data space and joint return period analysis, a detail discussion is given below.

Copula Data Space:Copula data space is a plot of joint probability isolines against drought duration and severity data values. This is studied for presenting joint probabilities values for different drought duration and severity combination. In copula data space one can observe how joint probability isolines behave for observed drought duration and severity data when different copula functions are employed.

Joint Return Period Analysis:Proper management of available water resources of drought prone area is needed to avoid severe water scarcity, for this planning of hydrological resources knowledge of return periods of drought and associated severities is important. The primary return period of drought event is return period for single variable i.e. return period with drought duration equal to or exceeding particular value of duration, return period with drought severity exceeding or equal to a particular value of severity. Primary return period is given in terms of expected inter-arrival time of drought and cumulative distribution of drought duration and severity, which is given as follows:

$$T_D=\frac{E(L)}{1-F_D(d)} \quad T_S=\frac{E(L)}{1-F_S(S)}$$

Where,$T_D$is the primary return period with a drought duration equal to or exceeding particular value of duration,$T_S$is the return period with a drought severity equal to or exceeding particular value of severity, L is theinter-arrival time of droughtwhich is defined as the time interval in between the start of two consecutive drought events, E (L) is the expected inter-arrival time of drought event, F_D (D) is the marginal probability distribution for drought duration and F_S (S)is the marginal probability distribution for drought severity.

As drought duration and severity are two interrelated properties of drought and has combined effect on final impact of drought, hence instead of primary return period, joint return period gives more important information for water resource planners as it considers both drought duration and severity values. Jointreturn period for duration and severity is given for drought duration equal to or exceeding particularvalue i.e. D≥d or drought severity equal to or exceedingparticular value i.e. S≥s.This joint return period of drought duration and severity is denotedby T_{DS} .

$$T_{DS}=\frac{E(L)}{P(D\geq d\,\text{or}\,S\geq s)}=\frac{E(L)}{1-C(F_D(d),F_S(s))}$$

Where,

$P(D \geq d \, or \, S \geq s)$ is probability distribution function, $C(F_D(d), F_S(s))$ is joint cumulative distribution function for drought duration and severity obtained using copula function.

RESULTS AND DISCUSSION

Bivariate drought analysis of drought variables using copula function is done for Marathwada region, Maharashtra state, India. There are total eight districts in Marathwada region which are - Hingoli, Jalna, Latur, Osmanabad, Parbhani, Aurangabad, Nanded, Beed. Marathwada region is having co-ordinates as, 70°5'East to 78°5'East in longitude whereas 17°5'North to 20°5'North in latitude. Refer figure 1 for a location map of the Marathwada region.

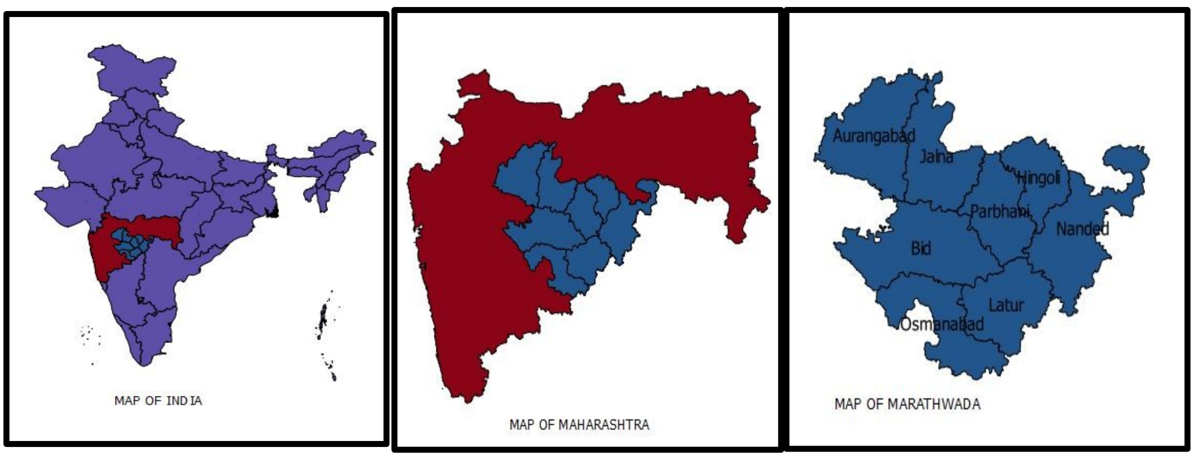


Fig.1: Location Map of Marathwada Region

Data Used

Monthly average precipitation for the whole Marathwada region is collected from IITM,Pune (Indian institute of tropical meteorology) for the time period of 117 years from 1900 to 2016. Monthly SPI-6 (6-month moving average) is calculated for 117 years and 87 drought events are identified, figure 2 shows SPI-6 series for Marathwada region. Using SPI 6 series, drought duration and severity data for 87 drought events is preparedand used for bivariate dependence analysis using copula function.Duration for which SPI-6 values are below "-1" is considered as drought duration. Summation of all SPI-6 values below "-1" within drought duration is taken as drought severity.

Dependence between Drought Variables

Drought is defined using characteristics such as drought duration and severity. Correlation statistics for duration and severity are provided in table 3. Spearman's rank correlation and Pearson's correlation coefficient are 0.88 and 0.87 respectively, which shows a strong correlation between drought duration and drought severity. All P values are less than 0.05, hence the correlations are significant at 5% (The P-value is the probability that if the correlation coefficient is '0', if this probability is less than the traditional 5% ($P<0.05$) then correlation coefficient is called statistically significant).

Table3: Correlation Statistics for Drought Duration and Severity

Correlation type	Correlation coefficient	P-value	Significant at 5 %
Kendall's tau	0.7510	0.000	Yes
Spearman's rho	0.8849	0.000	Yes
Pearson's coefficient	0.8737	0.000	Yes

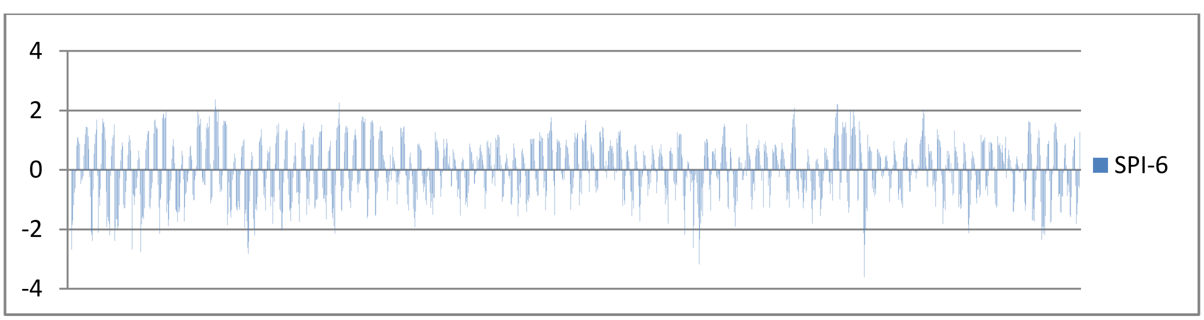


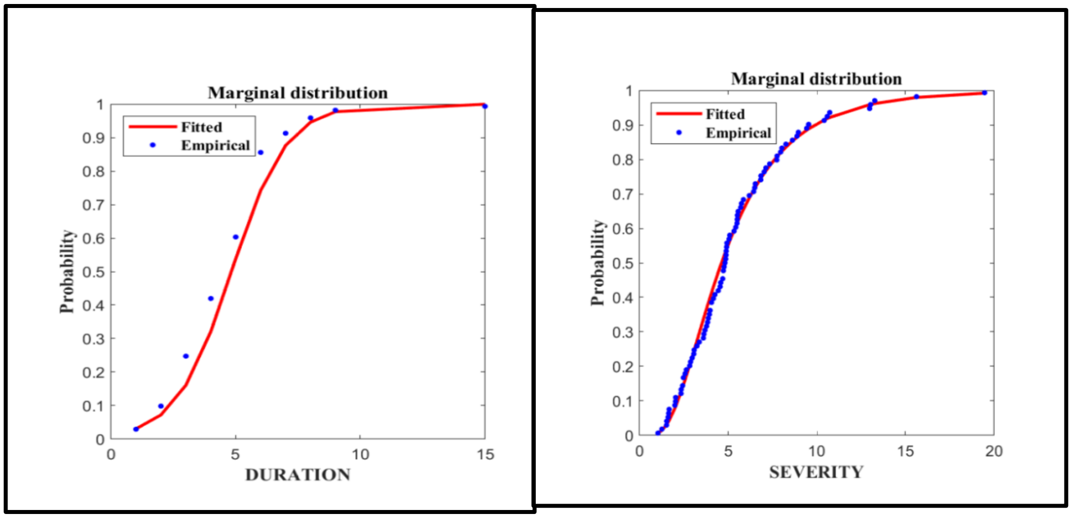
Fig. 2: Monthly SPI-6 for Marathwada Region from 1900 to 2016

Evaluation of Fitting of Marginal Distribution to Variable

Based on SPI-6 values drought duration and severity data is prepared and univariate fitting is tested for drought duration and severity. Best fitted distribution for drought duration is logistic distribution as shown in figure 3 (a) with parameter values as: μ = 4.829 and σ = 1.1063. Best fitted distribution for drought severity is Birnbaum-Saunders distribution as shown in figure 3 (b) with parameter values as: β = 4.598 and σ = 0.6156.

Copula Data Space

Plots of duration against severity forStudent t copula, Frank copula, AMH copula, Gumbel copula, Galambos copula and Clayton copulaare shown in figure 4 (a) to figure 4(f). All graphs from figure 4 (a) till figure4 (f) shows contour lines corresponding to values of joint probability of drought duration and severity (ranging from 0.1 to 0.9), all blue dots correspond to observed duration and severity pair, graph of observed drought duration against its individual probability is shown below contour plot and graph of observed drought severity against its individual probability is shown at left hand side of contour plot. As Galambos copula is previously selected as best fit copula function according to goodness of fit statistics, one can observe joint probability isolines for different combination of drought duration and severity in graph of Galambos copula function. These graphs provide important information of joint probability values for two variables. Joint probability values for best fitted and other selected copula functions can be observed from below graphs.



(a) (b)

Fig. 3: Univariate Fitting of Drought Duration and Severity.

Goodness of Fit Test for Selecting Best Fit Copula Function

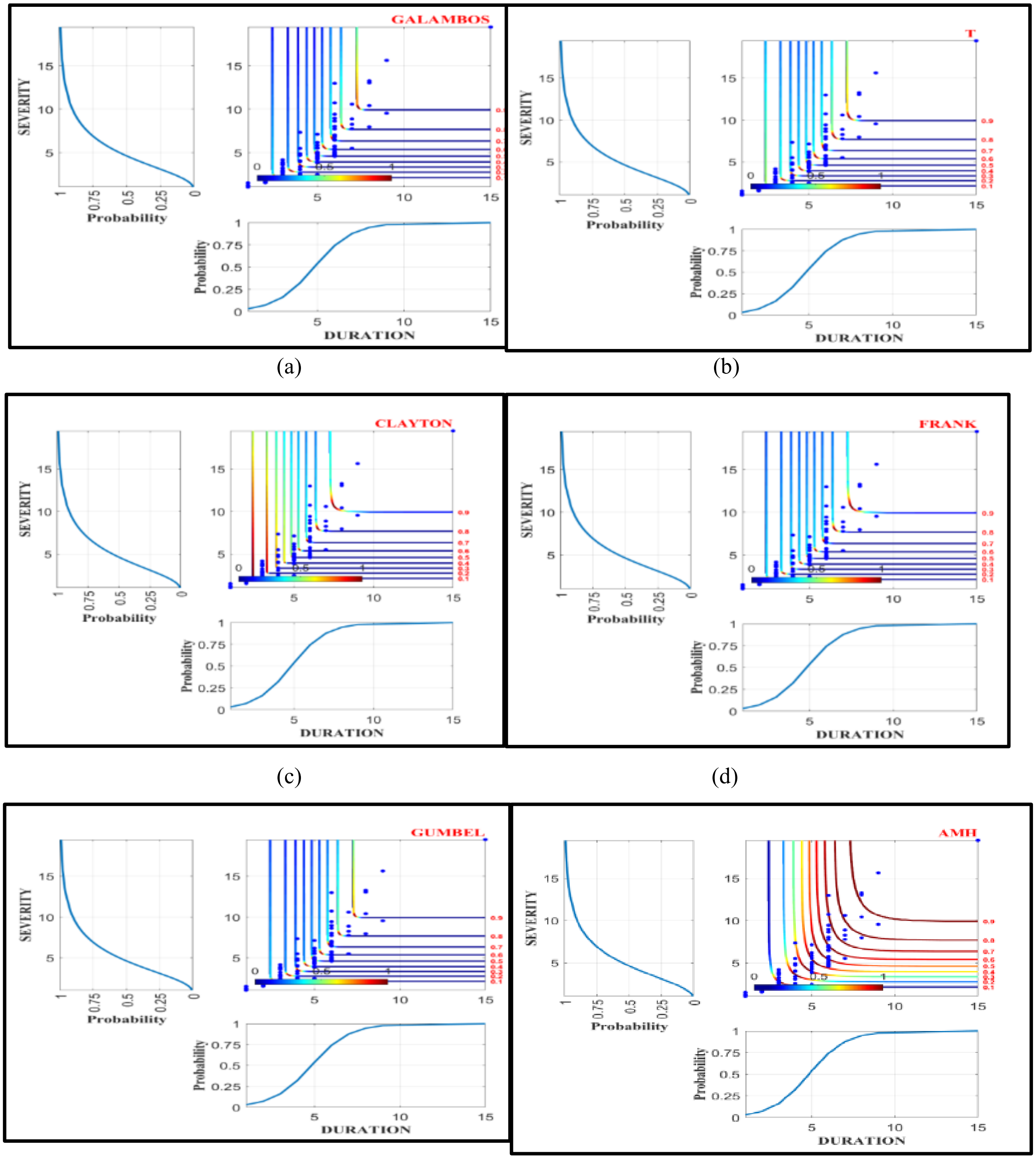
Various goodness of fit measures are taken into account to evaluate the best fit copula function. AIC (Akaike Information Criterion) is used to estimate the relative quality of a statistical model for a given set of data. AIC provides quality of each model with respect to another model. Copula models are ranked according to their AIC values. Copula model with lowest AIC value isranked one, table 5 shows ranking of copula functions according to AIC values.Copula function is selected one which is having highest AIC ranking(lowest AIC value).Table 5 shows parameter values for copula function. Student t copula is 2 parameter copula and all other copula functions are single parameter.

Return Period Analysis

The planning and management of water resources systems under drought conditions require the estimation of the joint return periods of drought events, which are characterized by high severities. Figure 6 is the graph of drought duration

Table 5: Goodness of Fit Statistics and Copula Parameter Values.

Copula Function	Rank by AIC	RMSE	NSE	Parameter 1	Parameter 2
Galambos	**1**	**0.3673**	0.9808	θ =7.6262	Not applicable
Gumbel	2	0.3674	0.9808	θ =2.8630	Not applicable
Student "t"	3	0.3672	0.9809	θ_1 = 0.8862	θ_2=10702011.7832
Frank	4	0.3746	0.9801	θ =10.9317	Not applicable
Clayton	5	0.3758	0.9800	θ =2.8694	Not applicable
AMH	6	0.9214	0.8795	θ =1.000	Not applicable



(a) (b) (c) (d) (e) (f)

Fig. 4: Copula Data Space for Galambos, Student t, Clayton, Frank, Gumbel and AMH Copula (Colour code of isoline is joint density levels with red colour representing higher densities and blue colour shows low density, blue dots are observed data points).

against severity along with joint return period contours for Galambos copula function which is best-fit copula function. Using contour lines of return period in figure6, the corresponding value of threshold duration on ‘x’ axis and threshold severity on ‘y’ axis is observed. These duration and severity values for all return period are mention in table 6. Values mentioned in table 6 are joint return period values of drought event with duration or severity equal to or exceeding the certain threshold value, e.g. for the return period of 2-year, drought duration exceeds or equals to 4.9 months or drought severity exceedsor equal to the value of 4.9.

CONCLUSION

Drought duration and severity is defined based on SPI-6 values calculated for time series precipitation data from

1900 till 2016for Marathwada region. The logistic and Birnbaum-Saunders distributions are the best fitted distributions forthe univariate fitting of drought duration and severity. Student 't' copula, Galambos copula, Clayton copula, Gumbel copula, Frank copula, and AMH copula functions are used to study dependence structure between drought duration and severity. AIC, RMSE, and NSE values are used to select the best-fit copula function. Galambos copula is found to be the best fit for time series data of drought duration andseverity for Marathwada region. Copula based joint probability values can be observed for different drought duration and severity combinations. Drought risk is estimated based on joint return period analysis which gives threshold drought duration and severity values for a particular return period which is important for water resource planning and management

Table 6: Drought Duration and Severity for Return Period

Sr. no.	Return period (years)	Drought duration (months)	Drought severity
1	2	4.9	4.9
2	5	6.3	7.5
3	10	7.3	10
4	25	8.3	13
5	50	9.1	15
6	100	9.9	17.5

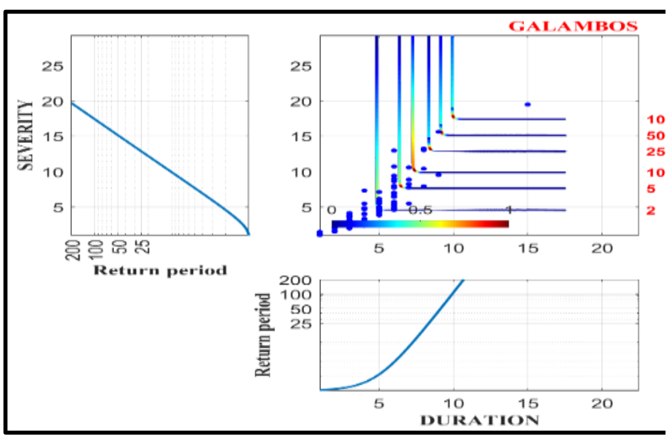


Fig 6: Joint Drought Return Period for Galambos Copula

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