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A SOLUTION TO THE GROUNDWATER INRVERSE PROBLEM, CONSIDERING A SYSTEM OF WELLS

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ABSTRACT

In this paper a new method is proposed, using genetic algorithms, to get the Transmissivity (T) and the storativity (S) of a confined aquifer, supposing that a system of wells pumps from the aquifer. Specifically, supposing that the following parameters are available: the hydraulic drawdowns at the location of one piezometer, the daily `volume of water extracted from each well and the locations of the neighbor wells, genetic algorithms can find the pumping flow rates of the wells, the pumping time schedules and the aquifer characteristics. The optimization is achieved creating populations of possible T, S and pumping time schedules, using Theis equation to conduct a transient flow analysis and calculate the hydraulic drawdowns at the piezometer.The examined populations are evaluated using the Root Mean Squared Error (RMSE) between the actual and the hypothetical drawdowns at the piezometer. Using this method, the aquifer characteristics can be found, without requiring neighbor wells to have stopped pumping when the measurements are conducted.

Keywords: *Groundwater, Inverse Problem, Genetic Algorithm, System of Wells, Storativity, Transmissivity*

INTRODUCTION

The popularity of groundwater resources management has radically increased the previous years. Groundwater extraction is now recognized as a part of climate change adaptation strategies and as an alternative to ensure water security and reduce water scarcity[1,2]. Identifying the aquifer characteristics, and mostly transmissivity (T) (or the equivalent permeability K) and storativity (S) is an essential requirement in order to calculate hydraulic drawdowns, estimate pollutants movement and generally guarantee safe and sufficient water resources. In general, solving the inverse problem consists of identifying the T and S values so that the simulated hydraulic drawdowns match the observed drawdowns[3]

The most popular method for estimating the characteristics of an aquifer is Cooper – Jacob's method, which can be used to get from drawdown time series T and S aquifer values[4]. Since 1946 a large number of papers has been published introducing advances on this method[5]or new methods that target on improving the results of Cooper-Jacob's method[6]. Most of the researchers target on estimating Transmissivity[7,8], some of whom take into consideration aquifer's characteristics spatial variability to increase accuracy[9–12].There is also a substantial amount of literature that deals with improving Storativity estimations[13,14]. Finally, it should be mentioned that it is often impossible to estimate aquifer characteristics using equations or numerical models (especially when examining fault zones, which can be highly heterogeneous structures). In some cases, ground's or groundwater's temperature measurements can be used to estimate groundwater flow paths[15,16].

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Genetic Algorithms are one of the most famous numerical methods in water resources management. Genetic Algorithms are probabilistic algorithms that mimic the functioning of natural phenomena, such as genetic inheritance and Darwinian struggle for survival[17]. Their applications in groundwater include pollutant's restriction and remediation [18,19], head level monitoring networks[20], and forecasting[21]. Considering the "inverse problem", Genetic Algorithms have been used, assuming steady flow conditions, to estimate the transmissivity of aquifers, dividing them into homogeneous sub-aquifers[3]. They have also been combined with BEM (boundary element method) in order to represent physical boundaries surrounding an aquifer with orwithout boundary conditions[22]. Dual reciprocity boundary element method has also been used to estimate transmissivities in a heterogeneous aquifers[23]. Genetic Algorithms have also been used to find aquifer characteristics using contaminants' concentration profiles, using as an objective function the difference between the produced concentration profiles and the actual pseudo-profiles[24].

In this paper genetic algorithms are used to estimate the aquifer characteristics, using the drawdown values obtained from one well – piezometer, while more than one wells pump water, affecting the aforementioned well. Theis equation[25]is used to model the transient flow conditions and calculate the hydraulic drawdowns obtained from the pumping wells. To test the Genetic Algorithms, pseudo-experimental data were used. In the objective function we choose to use root-mean-square error (RMSE), which is a classical statistical tool. Especially RMSE is widely used in combination with GA[21,26]. The methodology will be analytically described in the following chapters. We believe that it is an easy, applicable method, as the only pre-requirements for the model are the coordinates of the wells and the volume of water that each well pumps daily (regardless the duration of pumping or the pumping rates). As estimating water demand in irrigation becomes easier as geosciences evolve (assessing crops), the method proposed can be considered as an effective easy to use tool to estimate aquifer characteristics.

METHODOLOGY

Hydraulic Model

Suppose that a system of N wells is used to pump from an infinite confined aquifer. Then using the Theis equation[25] and the superposition principle, the hydraulic head level drawdown at point i can be defined as follows:

$$s_{i,\Delta t} = \frac{1}{4\pi T}\sum_{j=1}^{j=N} Q_j\, W(u_{i,j,\Delta t}) \qquad (1)$$

$$W(u_{i,j,\Delta t}) = -\gamma - \ln(u_{i,j,\Delta t}) - \sum_{k=1}^{\infty}\frac{(-1)^k (u_{i,j,\Delta t})^k}{k\, k!} \qquad (2)$$

$$u_{i,j,\Delta t} = \frac{S\, r_{i,j}^2}{4\, T\, \Delta t} \qquad (3)$$

In the above formulas, T represents aquifer's transmissivity, Q_j the pumping flow rate of well j, γ the Euler's constant, S the aquifer's storativity, $r_{i,j}$ the distance between point i and well j and Δt represents the duration of pumping. In this model we will divide time in minute sections. This way, if a well pumps for 2 hours, we will have 120 hydraulic drawdown values for the well considering s = f(t).

Supposing that the pumping flow rates are constant for a period Δt, they can be expressed using volumes as follows: $Q= dV/dt = V/\Delta t$. Supposing that we examine daily drawdowns, it is necessary that the volume of water extracted corresponds to the daily needs for water from the well.

The target of this paper is to use as less input data as possible to achieve accurate estimations, therefore the pumping flow rates and the pumping schedules (t_s, t_f) are not necessary, as long as volumes are available (m^3/day).

Optimization Process

The basic thought behind this model is described in figure 1. Supposing that a number of wells pumps water from an aquifer, there is a unique hydraulic drawdown curve $s_i(t)$ for each well that depends on the locations of the wells (x,y), the pumping schedules of each well (t_s stands for start and t_f stands for finish), the volumes of water extracted from each well (V) and the characteristics of an aquifer (T, S). "Pumping" can be conceptualized as a disturbance that is transmitted though the ground and arrives at the wells. The source of the disturbance can be represented using wells' characteristics (x, y, V, t_s, t_f). The well – piezometer that is used to measure the hydraulic drawdowns can be considered as the receiver of the disturbance. The aquifer acts like a conduit/filter between the source and the receiver, as changing aquifer's characteristics will result in different hydraulic drawdown curves at the receiver.

Genetic Algorithms are used to "change the filter" (aquifer characteristics T, S) so that the piezometer receives the same "message" with the one transmitted from the source. T and S are considered as variables. To insert T and S in genetic algorithms we used a binary chromosome of 28 digits length (14 for T and 14 for S), representing T values between $10^{-1}$and 10^{-5} (range: 10^4 values) and S values

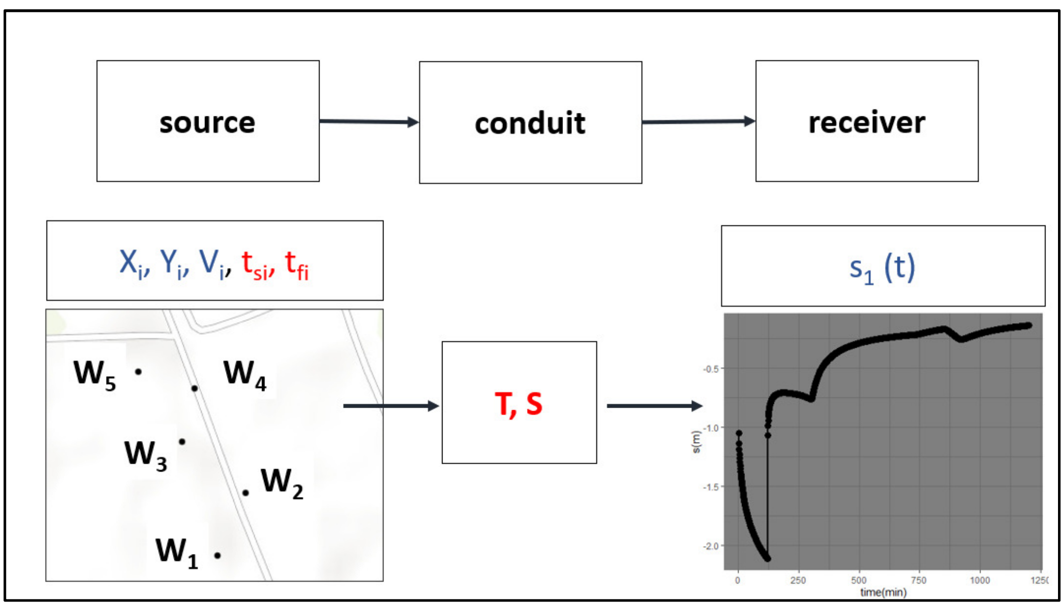

Fig.1: This flow chart represents the basic principles used in this paper. Blue colors represent necessary data parameters and Red colors represent the variables. For each set of well locations (x,y), water volumes extracted (V), pumping time schedules (t_s, t_f) and aquifer characteristics (T, S) we get a unique hydraulic drawdown curve at each well s(t). Supposing that one of these curves is known, and the coordinates of the wells and the water volumes extracted are available, the inverse problem can be solved by finding the parameters t_s, t_f, T, and S that create the hydraulic drawdown profile that is as close as possible to the hydraulic drawdown measurements.

between 10^{-3} and 10^{-7}(range: 10^4 values), which correspond to actual confined aquifers. It is necessary to use 14 binary digits to get 10^4 values. The time variables of each well t_s and t_f are represented using 11 bits for each that correspond to the duration of a day (1440 minutes).

The chromosomes of T, S, t_s, t_f are used to create populations and generations of solutions, following the mainstream processes of crossover, mutation and elitism. Specifically, in the following experiments 1500 generations were used with 150 populations each, crossover probability equal to 80%, mutation probability equal to 60% and elitism: 5 units. In the experiments conducted in this paper R package GA[27] was used.

2.3 Pseudo-Experimental Data and Tests

To test the aforementioned algorithm pseudo-experimental data were created, representing drawdown profiles for a well-piezometer. The genetic algorithms were used to find the characteristics of the aquifer and the time schedules that were used to create the pseudo-experimental data (figure 2).The root mean square error (RMSE) is used to estimate the difference between the actual and the hypothetical drawdowns. In the following relationship m represents the number of time sections examined, $s_{i\,(\text{chromosome})}$ represents the hydraulic drawdown which is a result of using a chromosome of (T, S, t_s, t_f) and $s_{i\,(observed)}$ represents the drawdown pseudo-data.

$$RMSE = \sqrt{\sum_{i=1}^{i=m} \frac{\left(s_{i\,(\text{chromosome})} - s_{i\,(observed)}\right)^2}{m}} \qquad (4)$$

APPLICATION - EXAMPLE

We examine 3 wells in a confined infinite aquifer with the following characteristics: $T=10^{-3}$ (m^2/s), $S=5*10^{-5}$and the following (random) distances (m): $r_{1,1}=r_{2,2}=r_{3,3}=r_0=0.2$, $r_{1,2}=r_{2,1}=158.47$, $r_{1,3}=r_{3,1}=143.91$, $r_{2,3}=r_{3,2}=67.84$,under the following 3 scenarios. In all of them well 3 is use as a piezometer and its hydraulic drawdowns are used as input data. The analysis is conducted using pseudo-data instead of real data, to avoid errors from measurements.

A. In the first case we suppose that only well 1 operates, for 6 hours, pumping in overall $V=216m^3$ (which corresponds to $q = 0.01m^3/sec$). This is a classical inverse problem, often solved using Cooper-Jacob equations.

B. In the second case we suppose that 2 wells operate simultaneously for 6 hours, with identical pumping rates ($V_1=V_2=216m^3$, $q=0.01m^3/sec$). This problem, seemingly similar to the previous one, cannot be solved using Cooper-Jacob, as pumping is a result of a system of wells.

C. Finally, we examine the most complicated problem. With the knowledge that $V_1=216m^3$ and $V_2=400m^3$,we use 10 hours of hydraulic drawdown measurements from piezometer (well 3, $V_3=0m^3$) to find out the aquifer characteristics, as well as the pumping schedules of wells 1 and 2 (when they started and when they stopped pumping).

Using the code to get the aquifer characteristics in the first scenario we get RMSE=0.005and the aquifer characteristic's results are the following: $T_A=0.99*10^{-3}$, $S_A=5.13*10^{-5}$. In this case we can see that the T and S values obtained are nearly identical to the actual characteristics ($T=10^{-3}$, $S=5*10^{-5}$). Similarly, the errors are minor when examining the second case, getting RMSE=0.011,$T_A=0.99*10^{-3}$and $S_B=5.14*10^{-5}$. Undoubtedly, C is the most interesting scenario. The Genetic Algorithm (figure 3) uses the hydraulic drawdowns obtained from the piezometer (figure 4 left), the pumping volumes and the locations of the wells, and finds the aquifer characteristics (T,S) and the pumping schedules (t_{s1}, t_{s2}, t_{f1}, t_{f2}). The pumping schedules used to create the piezometric curve are the following: W1: from 0:00 to 6:00, W2: from 4:00 to 9:00. Data were collected for W3: from 0:00 to 10:00.

The results provided by the genetic algorithm are the

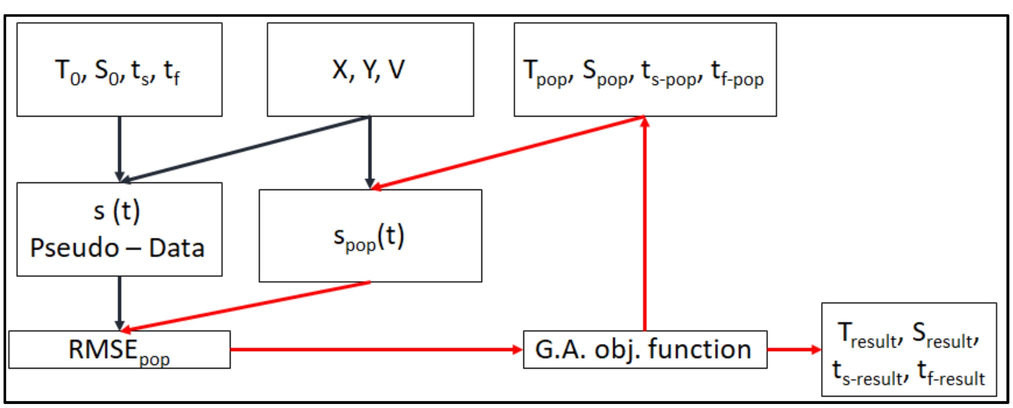

Fig.2: Flow chart representing the use of Pseudo Data in the tests conducted. The red lines represent the G.A. optimization process. Pseudo – Data can be replaced by actual hydraulic drawdown measurements. The drawdowns are used to test the hypothetical T, S, t_s, t_f created from the genetic algorithms, until the values that result to identical drawdowns are found (T_{result}, S_{result},$t_{s\text{-}result}$, $t_{f\text{-}result}$).

following: T_C=0.92*10^{-3}(m^2/s), S_C= 6.65*10^{-5}, W1 schedule: from 0:00 to 5:50, W2 schedule: from 3:59 to 8:58. The hydraulic drawdowns Root Mean Squared Error is RMSE=0.137.

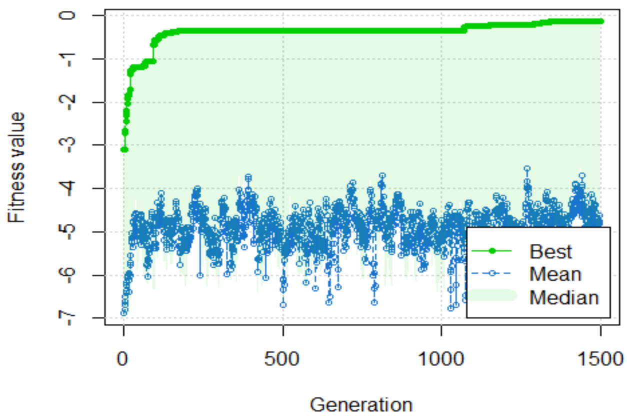

Fig.3: Genetic Algorithms' operation. The plot is automatically produced using G.A. R package[27]. X axis: generations, Y axis: -RMSE. Maximizing the (negative) RMSE is done using (T, S, t_s, t_f) that result in hydraulic drawdowns closer to the actual ones. "Best", "Mean" and "Median" represent populations. The best result creates the following figure.

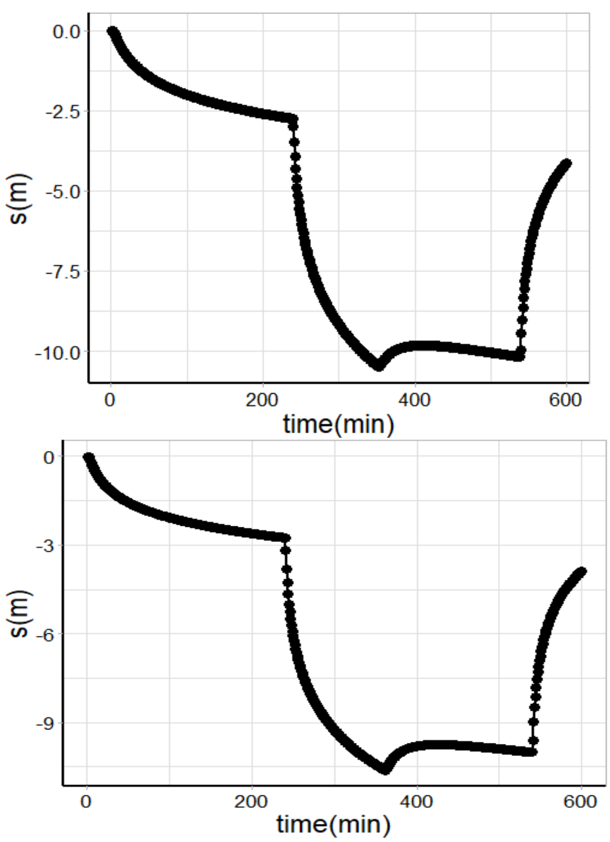

Fig.4: Left: pseudo-data hydraulic drawdowns (input). Right: Hydraulic drawdowns created minimizing RMSE, in order to find the aquifer characteristics (output). RMSE=0.137. Produced using ggplot[28]in R.The curves are nearly identical.

CONCLUSION

In this paper we examine the inverse problem of groundwater, in a confined aquifer, proposing a genetic algorithm that can be used to estimate the aquifer Transmissivity and Storativity using hydraulic drawdown measurements from one well. Moreover, the algorithm is capable of predicting the pumping schedule characteristics (e.g. the time that pumping started or stopped). The locations of the wells and a hydraulic drawdown profile are necessary to be inserted, as well as the pumping flow rates of the wells. The basic idea is that the aquifer can be considered as "conduit" transmitting a "distribution" (wells' pumping effect). Each aquifer results in a unique hydraulic drawdowns profile at a piezometer, which makes it possible to find the aquifer characteristics given the profile.

The algorithm has been tested in a limited number of cases, using pseudo-profiles to replace lack of real data and RMSE to test its accuracy. The results indicate that the algorithm can accurately solve the inverse problem. The code is available so that it can be used either in practice or to extend this research by examining more cases, testing the accuracy of the algorithm when the number of the wells increases, or when the aquifer characteristics change.

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