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CHARACTERISATION, TREND ASSESSMENT AND COPULA BASED BIVARIATE MODELLING OF METEOROLOGICAL DROUGHT FOR CENTRAL BRAHMAPUTRA VALLEY– AN AGRO CLIMATIC ZONE OF ASSAM

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ABSTRACT

This paper presents characterization, assessment of trend and bivariate modelling of droughts using copulas in Central Brahmaputra valley, an agro-climatic zone of Assam. Meteorological drought is modelled using Standardized Precipitation Index (SPI) at a time scale of 3, 6 and 12 months over 112 years during 1901-2012. For detection of any significant changes in the frequency of drought occurrences, trend analysis was performed using a nonparametric test (MK trend test) at a significance level of 0.05 and estimation of the magnitude of the temporal trend by Sen's slope approach and then dividing into for four different time periods. Further, drought is a complex phenomenon, characterizing severity, duration, and peak so, its univariate modeling may not capture complexity appropriately hence, probabilistic assessment of drought characteristics is investigated using copula method. For the construction of bivariate joint distributions Archimedean and Metaelliptical copulas were used. The drought risk was estimated using joint probabilities and return period for the study area.

Keywords*:* *Meteorological drought; Standardised Precipitation Index; Copula; Bivariate frequency analysis.*

INTRODUCTION

Drought is one of the several natural calamities, which build up slowly; unlike flood, no systematic method has yet been developed either for their complete understanding or their prediction (Sales, 1986). Type of drought varies from place to place, depending upon the climate of the India (Tiwari *et al*., 2007). Investigations are going on to find out a suitable approach for the identification of drought (Shrivastava *et al*., 2008). This is usually done by comparing the current situation to the historical average, often based on a 30-year period of record. Thornthwaite (1948) classified droughts in three types: a) the permanent droughts of the driest climates, b) the seasonal droughts in climate having well defined wet and dry seasons generally in tropical areas, and c) the contingent droughts resulting from irregular and variable occurrence of rainfall in areas with normally sufficient rainfall. Drought is also classified into four types are: meteorological (lack of precipitation), agricultural, (lack of moisture in the soil where crops grow), hydrological (low levels of water in lakes and reservoirs and socio-economic (failure of water resources systems to meet water demands).

The drought has three essential characteristics i.e., intensity/severity, duration, and frequency. For a given region, if these attributes are specified or known, then the planning for drought mitigation would be relatively easy (Pandey and Ramasastri, 2001). For drought events, the commonly used time limits are months, followed by season and year. Drought events are easily detected from a continuous series of monthly, seasonal or yearly data (Pandey *et al*., 2008).

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Drought is a complex phenomenon, so, its univariate modeling may not capture complexity appropriately. A separate analysis of drought duration distribution and drought severity distributions cannot reveal the significant correlation between them. Quite often the study of the return period involves univariate cases, but this may lead to an under/overestimation of the risk. Indeed, many hydrological events are characterized by the joint behavior of several random variables (RVs), and these are usually nondependent. So, instead of using traditional univariate analysis for drought assessment, a better approach for describing drought characteristics is to derive the joint distribution of drought variables (Mishra and Singh, 2010). Shiau and Shen (2001), Bonaccorso *et al.* (2003), Kim *et al.* (2003), González and Valdés (2003), Salas *et al.* (2005), and Cancelliere and Salas (2004) proposed different methods to investigate the joint distribution of drought duration and drought severity or intensity. A bivariate distribution is thus more common and easier for describing the correlated hydrologic variables. These bivariate distributions have either complex mathematical derivations or their parameters are obtained by fitting the observed or generated data (Shiau, 2006). Several methods have been proposed to investigate the bivariate characteristics of droughts. Shiau and Shen (2001), Gonz´alez and Vlad´ es (2003), Salas *et al.* (2005), and Mishra *et al.* (2009) used the product of the conditional distribution of drought severity for a given drought duration and the marginal distribution of drought duration to construct the joint distribution of drought duration and severity. Kim *et al.* (2003) used a nonparametric bivariate kernel estimator to establish the joint distribution of droughts.

Meteorological drought characterization is important to know the risk related agricultural production. Drought is a common phenomenon in the arid region of India. In the humid region of northeast India lack of rainfall in the pre-monsoon season (March to May) and the post-monsoon period (November to February) leads to drought condition. The scanty studies are available on the characterisation of drought for the humid region of northeast India. Hence, in this study, an attempt has been made for characterization, trend assessment and copula-based bivariate modeling of meteorological drought for central Brahmaputra valley agro-climatic zone of Assam.

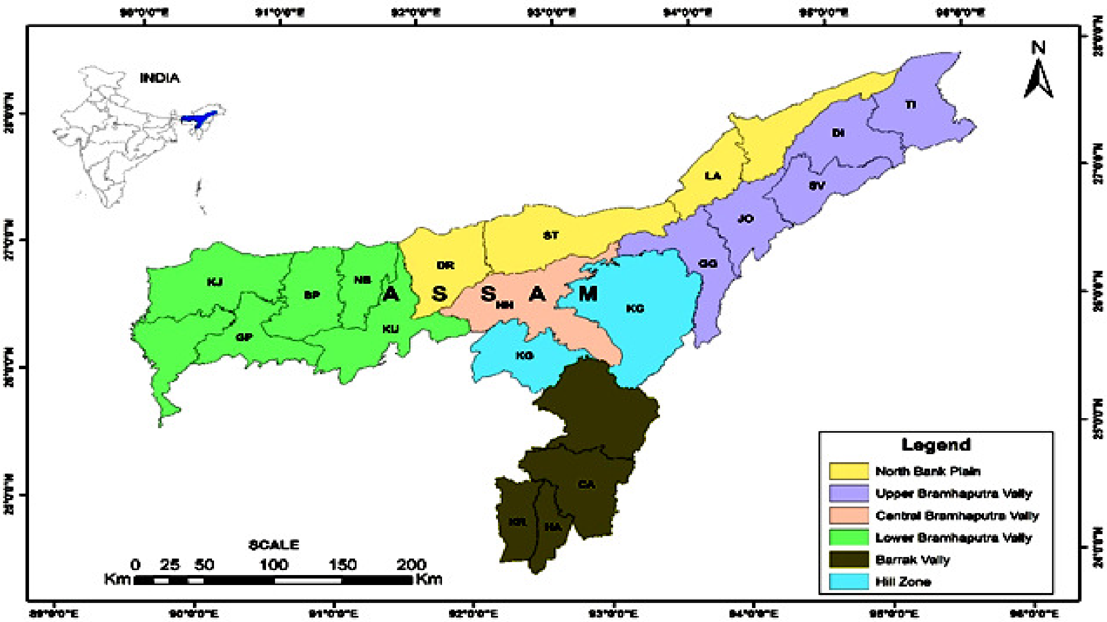
MATERIALS AND METHODS

Study area and acquisition of rainfall data

The state of Assam is situated between 24^0 05' N and 27^0 55'N latitudes and 89 ^{0}45' E and 96^0 E, having 32 districts with an area of 78438 km^2 and covers 2.4 per cent of the country in area. There are 06 agro-climatic zones in Assam. Central Brahmaputra valley - Agro-climatic zone of Assam consists of Nagaon and Morigaon districts (Fig.1). Nagaon is bounded on the north by the Sonitpur district and the Brahmaputra River. On the south, it borders the western Karbi Anglong District and Dima Hasao. On the east, it is bounded by eastern Karbi Anglong and the Golaghat district, while on the west it neighbors the Marigaon district. Nagaon has a tropical climate.The rainfall in Nagaon is much lesser in winter than in summer. This climate is considered to be Aw according to the Köppen-Geiger climate classification. The average annual temperature is 26.5°C in Nagaon and its rainfall averages 2401 mm. Monthly rainfall data of Nagaon was collected from IMD from the year 1901 to 2012. Rainfall data of Morigaon were discontinued for many years, hence they were not considered for the study. The climate of Central Brahmaputra valley agro-climatic zone of Assam is classified as Pre-monsoon (March-May), Monsoon (June to October) and Post-monsoon (November to February).

SPI for drought analysis and its computation

SPI represents the wet or dryness state. For SPI calculation normally the long-term precipitation record of 30 years or more is required. This long-term record is fitted to the gamma probability distribution function, which is then transformed to normal distribution so that the mean SPI for the location and



Meteorological Stations (abbrevations): Cachar (CA), Hailakandi (HA), Karimganj (KR), Nagaon (NN), Karbi Anglong (KG), Dima Hasao (North Cachar Hills) (NC), Kamrup (KU), Goalpara (GP), Kokrajhar (KJ), Barpeta (BP), Nalbari (NB), Dhubri (DU), Lakhimpur (LA), Sonitpur (ST), Darrang (DR), Dhemaji (DM), Tinsukia (TI), Dibrugargh (DI), Jorhat (JO), Sivasagar (SV), Golaghat (GG)

Fig. 1: Location map of the study area

Table 1: Drought classification based on SPI value and its corresponding probabilities

SPI value	Category	Probability
2.00 or more	Extremely wet	0.977-1.000
1.50 to 1.99	Severely wet	0.933-0.977
1.00 to 1.49	Moderately wet	0.841-0.933
-0.99 to 0.99	Near Normal	0.159-0.841
-1.00 to -1.49	Moderately dry	0.023-0.159
-1.50 to -1.99	Severely dry	0.023-0.067
-2 or less	Extremely dry	0.000-0.023

(Source: McKee *et al.,* 1993)

the desired period is zero. Positive SPI values indicate greater than median precipitation while negative values indicate less than median precipitation. Mishra and Desai (2005) have given the details of SPI. The SPI was computed by fitting gamma probability density function to the frequency distribution of precipitation summed over the time scale for the desired period. This was performed separately for different time scales (3, 6 and 12 months). Once standardized, drought classification described by McKee *et al.* (1993) was used as presented in Table 1.

Identification of drought was determined using SPI series by assuming a drought period as a consecutive number of time intervals where SPI values are less than 0 (Shiau, 2006). Drought duration(D), severity(S), interval time(L) was determined as per definition is given by Shiau(2006), Mishra and Singh(2010) and Song and Singh (2010a) respectively.

In this study, identification of drought and its variable was determined at different SPI time scale (3 months, 6 months and 12 months) for six for Central Brahmaputra agro-climatic zone.

Drought trend identification using non-parametric tests

For detection of any significant changes in the frequency of drought occurrences, trend analysis was performed on SPI-3, SPI-6 and SPI-12-time series using rank-based nonparametric Mann-Kendall [Mann,1945; Kendall 1975] method. If the value of MK test statistic (Z) lies within limits ± 1.96, then the null hypothesis of no trend can be accepted at the 5% level of significance. Otherwise, the null hypothesis can be rejected and the alternative hypothesis can be accepted at the 5% level of significance (Jhajharia *et al.,* 2014). For estimation of the magnitude of trends in the time series data, Sen's non-parametric method (Sen 1968) was used. The Sen's slope is estimated as the median of all pair-wise slopes between each pair of points in the data set (Thiel,1950: Sen,1968: Helsel and Hirsch,2002). Positive Sen's slope indicates rising trend while negative Sen's slope indicates falling trend.

Trend was analyzed for four different time periods: (i) long-term trend was computed for the complete 112 years time series (1901-2012); (ii) short-term trend was computed for three cases: 43 years (1901-1943), 35 years (1944-1978) and 34 years (1979-2012)

Dependence measures

Pearson's linear correlation and two nonparametric dependence measures—Spearman's rank correlation ρ and Kendall's τ were estimated along with their p- values for analyzing the dependence among the drought variables at different SPI time scale.

Univariate probability distribution fit of drought variables

Normal, Lognormal, Gamma and Weibull distributions were used to fit different drought variables. Using maximum likelihood (ML) method the parameters for univariate distributions were estimated. edBas on the highest value of the log-likelihood function: and the lowest value of AIC and BIC the best-fitted distribution for every drought variables were selected.

Testing the goodness of fit

The Akaike's information criteria (AIC) (Akaike 1974; Bozdogan 2000) and the Bayesian information criterion (BIC) was employed to compute the goodness-of-fit measures for fitting univariate distribution for each drought variable. For this criterion, the model which has minimum value will be chosen as the best model. The method of maximum likelihood was also used for fitting drought variables, and the best value of a parameter should be that value which maximizes the likelihood or joint probability of occurrence of the observed sample.

About copula

Theory of copulas is based on Copulas Sklar theorem (Sklar,1959).In multivariate case it is represented as follows :

$$G(X) = C\{F_1(x_1),\dots,F_d(x_d);\theta\} \qquad X \in R^d \qquad (1)$$

Where, F1….Fd., are the continuous marginal CDFs the function and C:[0,1]^d → [0,1] is described as d- dimensional copula.For further details of the copula, work of Nelson (2006) and Joe (1997) can be referred.

Goodness of fit tests for copulas

The Akaike's information criterion(AIC) (Akaike,1974; Bozdogan, 2000) and Schwarz information criterion(SIC) (Neath and Cavanaught,1997) were employed to compute the goodness-of-fit measures between fitted copula and empirical joint distribution. For both the criteria the model which has the minimum value was chosen as the best fit model.

Table 2: Copula functions and parameters space of the considered bivariate copulas

Copula Family	Copula type	$C_\theta(U,V)$
Archemedian	Clayton	$\left(U^{-\theta}+V^{-\theta}-1\right)^{-1/\theta}, \theta \geq 0$
	Frank	$-\frac{1}{\theta}\ ln\left[1+\frac{\left(e^{-\theta U}-1\right)\left(e^{-\theta V}-1\right)}{\left(e^{-\theta}-1\right)}\right], \theta \neq 0$
	Gumbel	$exp\left[-\left\{(-\ln U)^{\theta}+(-\ln V)^{\theta}\right\}^{1/\theta}\right], \theta > 1$
Metaelliptical	Normal	$\int_{-\infty}^{\phi^{-1}(U)}\int_{-\infty}^{\phi^{-1}(V)}\frac{1}{2\pi(1-\rho^2)^{\frac{1}{2}}}exp\left\{-\frac{(U^2-2\rho UV+V^2)}{2(1-\rho^2)}\right\}dUdV$ Where ρ = linear correlation coefficient
	Student's t	$\int_{-\infty}^{t_v^{-1}(U)}\int_{-\infty}^{t_v^{-1}(V)}\frac{1}{2\pi(1-\rho^2)^{\frac{1}{2}}}\left[1+\frac{U^2-2\rho UV+V^2}{v(1-\rho^2)}\right]^{-\frac{(v+2)}{2}}$ Where, v = degree of freedom, ρ = linear correlation function

Bivariate modeling of drought using copula

To find out the best-fit copula for modeling, two-dimensional Archimedean and meta-elliptical copulas was carried out among three drought characteristics. The different Archimedean and meta-elliptical copulas are given in Table 2.

Assessment of tail dependence of fitted copula model

In the bivariate frequency analysis of drought, if tail dependence structure amongst drought variables is not well retained by a chosen copula, it will provide a high uncertainty in the estimation of extreme values. Hence, the assessment of tail dependence plays an important role in evaluating the adequacy of the selected copula family. Copula-based upper tail (λ_u) and lower tail (λ_L) coefficients for different copulas are given in Table 3.

Table 3: Tail dependence coefficients for different copulas

Copula	λ_u	λ_u^{CFG}
Clayton	0	0
Frank	0	0
Gumbel	$2-2^{1/\theta}$	$2^{-1/\theta}$
Normal	0	0
Student's t	$2-2t_{\nu+1}\left(\sqrt{\nu+1}\sqrt{1-\rho}/\sqrt{1+\rho}\right)$	

where, ν denotes the degrees of freedom, t_ν (z) the value of the t-distribution function with ν degrees of freedom at point z and ν is the correlation parameter (which is for ν > 2 the coefficient of linear correlation).

In the analysis of the extreme event, the upper tail dependence plays a greater role than the lower tail dependence. Therefore, in this study, we focused on the upper tail dependence(λ_u). The empirical upper tail coefficient (λ_u^{CFG}), suggested by Frahm *et al.*, 2005 was estimated using the following expression.

$$\lambda_u^{CFG} = 2 - 2.\exp\left[\frac{1}{n}\sum_{i=1}^{n} log\left\{\frac{\sqrt{\log\frac{1}{u_i}\log\frac{1}{v_i}}}{\log\frac{1}{max(u_i,v_i)^2}}\right\}\right] \quad (2)$$

Where u_i and v_i are the CPF's of the drought characteristics taken up in this study.

As per Table 3, Clayton, Frank, Normal copulas do not have upper tail dependence$(\lambda_u = 0)$. For the rest of copulas, upper tail λ_u and empirical upper tail coefficient (λ_u^{CFG}) were compared to verify the adequacy of the copula.

Estimation of return period for drought events

Univariate return period

As per concept of Shiau and Shen (2001), the return period for droughts variables i.e. drought duration, drought severity and drought interval were estimated from the mathematical expression given below:

$$T_D = \frac{E(L)}{1-F_D(d)};= \frac{E(L)}{1-F_S(s)};T_{L_d} = \frac{E(L)}{1-F_L(l)} \quad (3)$$

Where, $E(L)$is the expected drought interval time, F_D is the cumulative drought duration distribution function, F_S is the cumulative drought severity distribution function and F_L is the cumulative drought interval distribution function.

Bivariate and trivariate return period

As per concept of Shiau (2003), the joint drought duration and severity return period, severity and drought interval and length and duration return periods T^{and} and T^{or} were calculated using the formula described below:

$$T_{DS}^{and} = \frac{E(L)}{1-F_D(d)-F_S(s)+C(F_D(d),F_S(s))} \quad (4)$$

$$T_{SL}^{and} = \frac{E(L)}{1-F_S(s)-F_L(l)+C(F_S(s),F_L(l))} \quad (5)$$

$$T_{LD}^{and} = \frac{E(L)}{1-F_L(l)-F_D(d)+C(F_L(l),F_D(d))} \quad (6)$$

$$T_{DS}^{or} = \frac{E(L)}{1-C(F_D(d),F_S(s))} \quad (7)$$

$$T_{SL}^{or} = \frac{E(L)}{1-C(F_S(s),F_L(l))} \quad (8)$$

$$T_{LD}^{or} = \frac{E(L)}{1-C(F_L(l),F_D(d))} \quad (9)$$

Where, T_{DS}^{and} is the joint return period for D ≥ d and S ≥ s ; T_{SL}^{and} is the joint return period for S ≥ s and $L \geq l$; T_{LD}^{and} is the joint return period for $L \geq l$ and D ≥ d ; T_{DS}^{or} is the joint return period for D ≥ d or S ≥ s ; T_{SL}^{or} is the joint return period for S ≥ s or $L \geq l$; T_{LD}^{or} is the joint return period for $L \geq l$ or D ≥ d.

RESULTS AND DISCUSSION

Drought analysis using standardized precipitation index (SPI) for central Brahmaputra valley –an agro-climatic zone of Assam

Figs. 2 and 3 represent the drought analysis using SPI on 3 months time scale. The moderately dry drought was observed during the months of January-March, April-June, July-September, October-December in 9, 3, 2, 14 number of years respectively out of 112 observed years.

The severely dry drought was observed during the months of January-March, April-June, July-September, October-December in 6, 1, 2, 2 number of years respectively out of 112 observed years.

The extremely dry drought was observed during the months of January-March, April-June, July-September, October-December in 4, 8, 8, 3 number of years respectively out of 112 observed years.

The range of drought duration, severity, and drought length was found as 3 to 21 months, 1.06 to 16.75, 6 to 207 months respectively. The maximum drought severity (16.47) was observed for the year July 2009 to September 2009. The minimum drought severity (1.06) was witnessed during January 1975 to March 1975.

Drought analysis using SPI on 6 months time scale

Figs.4 and 5 present the drought analysis using SPI on 6 months time scale. The moderately dry drought was observed during the months of January-June, July-December in 3, 2 number of years respectively out of 112 observed years.

The severely dry drought was observed during the months of January-June, July-December in 2, 2 number of years respectively out of 112 observed years.

The extremely dry drought was observed during the months of

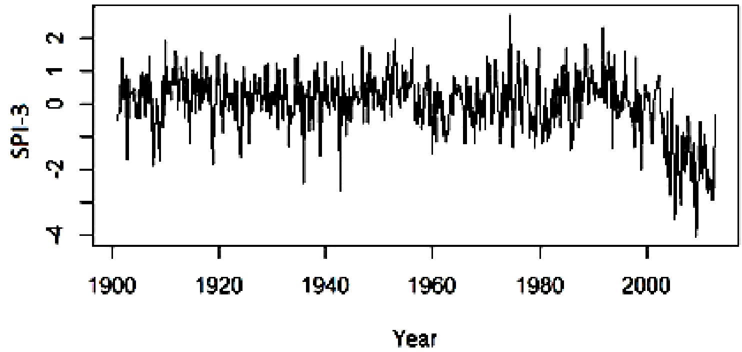


Fig.2: SPI for 3-months time scale

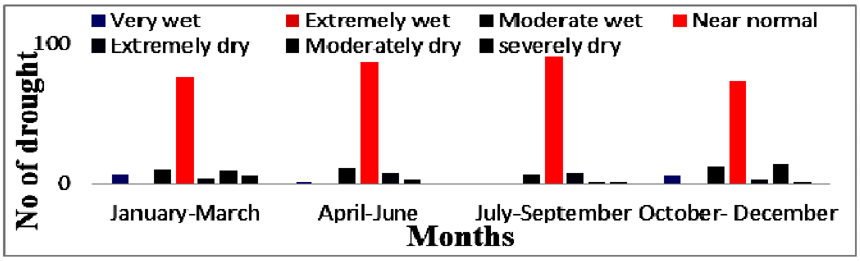


Fig. 3: Occurrence of drought at 3-months time scale

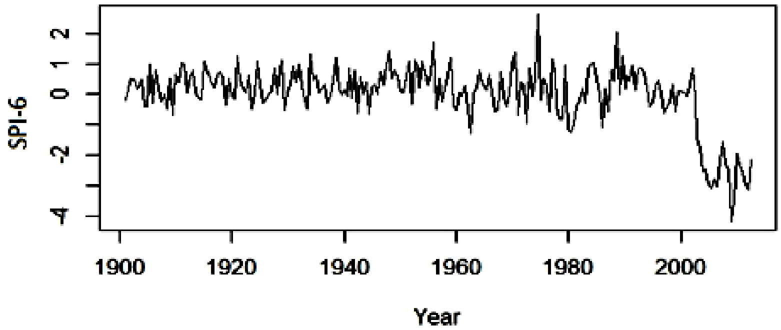


Fig. 4: SPI for 6 months time scale

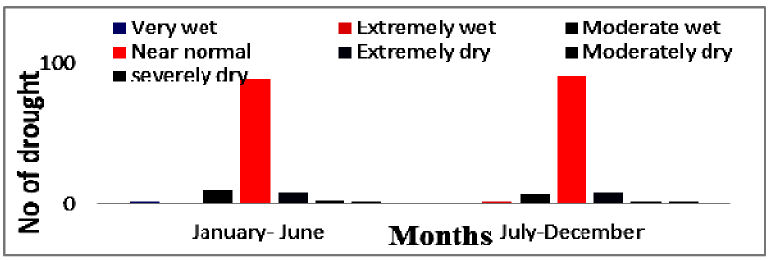


Fig 5: Occurrence of drought at 6 months time scale

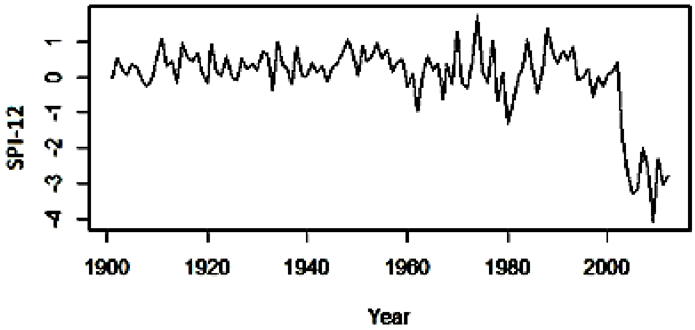


Fig. 6: SPI for 12 months time scale

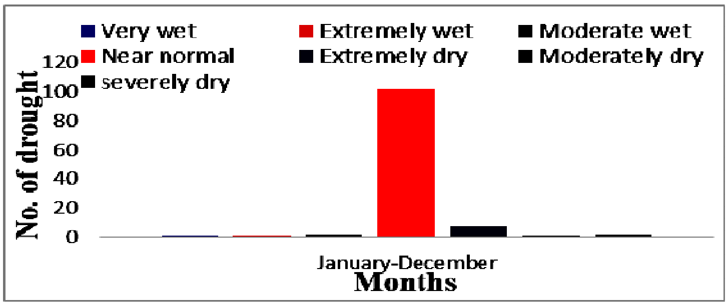


Fig.7: Occurrence of drought at 12 months time scale

January-June, July-December in 8, 8 number of years respectively out of 112 observed years.

The range of drought duration, severity, and drought length was found as 6 to 120 months, 1.05 to 51.09, 2 to 216 months respectively. The maximum drought severity (51.09) was observed for the year July 2003 to December 2012. The minimum drought severity (1.05) was observed during January 1986 to June, 1986.

Drought analysis using SPI on 12 months time scale

Figs. 6 and 7present the drought analysis using SPI on 12 months time scale. During January-December, moderately dry drought was observed in 1 year, severely dry drought in 2 years and extremely dry drought in 8 years out of 112 observed years.

The range of drought duration, severity, and drought length was found as 12 to 120 months, 1.3 to 2.77, 0 to 276 months respectively. The maximum drought severity (2.77) was observed during January 2012 to December 2012. The minimum drought severity (1.3) was observed from January 1980 to December 1980.

Analysis of Drought Trend

Table 4 shows the results of trend analysis at different SPI time scale. In all the cases, i.e., SPI-3, SPI-6 and SPI-12 time series, a significant falling trend was observed for the entire time period (1901-2013) and short-term trends time period (1979-2013). No trend at 5 % significant level was observed during the short-term time period (1901-1943) and short-term time period (1944-1978) in all the cases. The falling trend of SPI time series in the above mentioned time period indicated an increase in the number of drought occurrences.

Table 4: Mann-Kendall test statistics at magnitude Sen's slope parameters

Time Scale	Time Period	Z_{MK}	b	Trend
3 months	1901-2013	**-5.37**	-0.002	Falling
	1901-1943	-0.87	-0.001	
	1944-1978	-1.81	-0.003	
	1979-2013	**-5.94**	-0.017	Falling
6 months	1901-2013	**-4.63**	-0.004	Falling
	1901-1943	-0.36	-0.001	
	1944-1978	-1.81	-0.008	
	1979-2013	**-5.10**	-0.05	Falling
12 months	1901-2013	**-3.27**	-0.007	Falling
	1901-1943	-0.06	-0.002	
	1944-1978	-1.82	-0.022	
	1979-2013	**-3.48**	-0.105	Falling

(Z_{MK} and b denote Mann-Kendall test statistics and slope estimate using Sen'S slope estimator respectively. Bold members indicate that trend is significant at 5% significant level.)

Univariate and Bivariate Frequency analysis of drought

Dependence Measures of the drought variables

Table 5 presents the qualitative dependence among the drought variables using Pearson's linear correlation for different time scales. Result confirmed that for drought events all variables showed positive association and a highly correlated relationship were also observed. Some drought variables showed a small negative relation but it was small and close to 0, which means that fully nested Archimedean copula can be used in these cases.

Table 5: Correlation among drought variables

Time scale	Dependence Measure	Duration-Severity	Duration-Interval	Severity- Interval
3 Months	Pearson’s r	-0.266(0.1115)	0.977(2.2e-16)	-0.260(0.1191)
	Spearman’s ρ	-0.2695(0.1066)	0.835(1.281e-10)	-0.3647(0.02646)
	Kendall's τ	-0.1503(0.2018)	0.7214(5.535e-08)	-0.2973(0.0285)
6 Months	Pearson’s r	0.376(0.5317)	0.999(1.225e-05)	0.388(0.5182)
	Spearman’s ρ	0.5(0.45)	0.894(0.040)	0.223(0.717)
	Kendall's τ	0.2(0.8167)	0.836(0.0522)	0.1195(0.7815)
12 Months	Pearson’s r	0.978(0.021)	0.982(0.017)	0.976(0.023)
	Spearman’s ρ	0.632(0.3675)	0.774(0.2254)	0.816(0.183)
	Kendall's τ	0.547(0.278)	0.774(0.1573)	0.707(0.1797)

(Parameter in parenthesis indicates p-value)

Table 6: Estimated parameters and comparison of performance of marginal distribution fit of drought variables

SPI Time Scale	Drought Variable	Distribution	Parameters				Maximum Likelihood Value	AIC	BIC
			Mean	SD	Shape	Scale			
3 months	Severity	Normal	3.10	13.60			-100.20	204.43	207.65
		Log Normal	2.90	6.10			**-69.90**	**143.83**	**147.05**
		Gamma	3.10	6.20	1.60	2.00	-77.10	158.28	161.50
		Weibull	3.10	8.00	3.30	1.10	-78.60	161.19	164.42
	Duration	Normal	5.10	18.90			-106.40	216.80	220.02
		Log Normal	4.90	8.30			**-82.90**	**169.71**	**172.93**
		Gamma	5.10	9.40	2.80	1.80	-89.10	182.25	185.48
		Weibull	5.20	13.30	5.70	1.40	-92.90	189.90	193.12
	Interval	Normal	11.20	159.70			-145.90	295.69	298.91
		Log Normal	13.20	548.10			-127.70	259.37	262.60
		Gamma	11.20	119.00	1.10	10.60	**-126.30**	**256.66**	**259.89**
		Weibull	11.20	125.40	11.20	1.00	-126.40	256.72	259.95
6 months	Severity	Normal	11.50	491.30			-22.10	204.43	207.65
		Log Normal	11.00	1527.30			**-14.50**	**143.83**	**147.05**
		Gamma	11.50	275.30	0.50	24.00	-15.90	158.28	161.50
		Weibull	10.20				-15.50	161.19	164.42
	Duration	Normal	29.60	1896.20			-25.40	56.27	55.49
		Log Normal	29.10	3715.30			**-20.50**	**44.97**	**44.19**
		Gamma	30.00	1296.60	0.70	43.20	-21.80	47.50	46.72
		Weibull	28.60	1515.30	23.90	0.70	-21.50	47.03	46.25
	Interval	Normal	139.20	9954.20			-29.60	204.43	207.65
		Log Normal	9213.30	5.54E+12			-30.50	143.83	147.05
		Gamma	139.20	40872.00	0.50	293.60	**-28.40**	**158.28**	**161.50**
		Weibull	167.80	76873.60	118.80	0.60	-29.00	161.19	164.42
12 months	Severity	Normal	2.00	1.00			-2.30	-105.59	-101.50
		Log Normal	2.10	1.40			-2.30	-122.50	-118.42
		Gamma	2.00	0.50	7.80	0.30	**-2.10**	**-116.93**	**-112.84**
		Weibull	2.00	0.50	2.20	3.30	-2.10	-68.89	-64.80
	Duration	Normal	66.00	5832.00			-11.00	46.11	44.89
		Log Normal	142.80	2.69E+05			-10.58	41.23	40.01
		Gamma	66.00	4195.00	1.00	63.60	-10.37	41.30	40.08
		Weibull	66.00	4009.70	67.10	1.00	**-10.37**	**39.81**	**38.58**
	Interval	Normal	150.10	44970.00			-13.10	-105.59	-101.50
		Log Normal	4.99E+07	2.07E+29			-9.20	-116.93	-112.84
		Gamma	150.10	102744.00	0.20	684.70	**-8.80**	**-122.50**	**-118.42**
		Weibull	369.20	4.00E+06	39.70	0.30	-9.00	-68.89	-64.80

Univariate modeling of Drought Variables

For drought variables i.e., severity, duration log-normal probability distribution functions were found the best fit at 3 months and 6 months time scale while gamma probability distribution functions for 12 months time scale. For drought interval, gamma distribution was found the best fit for SPI at 3, 6 and 12 months time scale (Table 6).

Table 7: Estimated copula parameters and comparison of performance of different copula families

SPI Time scale	Copula type	Copula Family	Parameters	AIC	SIC	BF
3 months	Bivariate(D/S)	Clayton	□ =5.157	47.4	44.5	
		Frank	□ =15.871	35.4	33.1	
		Gumbel	□ =4.0972	43.7	40.8	
		Normal	ρ =0.9405	**-43.5**	**-45.0**	**BF**
		T	ν =5.44 ,ρ =0.942	10.9	8.0	
	Bivariate(S/L)	Clayton	□ =1.5E-06	-0.2	-3.1	
		Frank	□ =-2.661	**-1.8**	**-4.7**	**BF**
		Gumbel	□ =1.00	-1.0	-3.9	
		Normal	ρ =-0.33	0.3	-1.2	
		T	ν =32.03, ρ =-0.341	-2.0	-4.9	
	Bivariate(L/D)	Clayton	□ =1.5E-06	0.9	-1.9	
		Frank	□ =-2.661311357	**0.5**	**-2.4**	**BF**
		Gumbel	□ =1.00	0.9	-2.0	
		Normal	ρ =-0.3081	1.9	0.4	
		T	ν =32.03, ρ =-0.341	0.2	-2.6	*
6 months	Bivariate(D/S)	Clayton	□ =16.564	-0.2	6.6	
		Frank	□ =16.564	-2.2	4.6	
		Gumbel	□ =18.5653	-1.3	5.5	
		Normal	ρ =0.9955	**-520.7**	**-519.0**	**BF**
		T	ν =1.58E+07 ,ρ =0.996	-11.5	-4.7	
	Bivariate(S/L)	Clayton	□ =0.3007	-9.0	-2.2	
		Frank	□ =0.30074	**-9.1**	**-2.4**	**BF**
		Gumbel	□ =1.222755483	-9.1	2.3	
		Normal	ρ =0.392739089	-2.4	-0.7	
		T	ν =1.4E+07, ρ =0.397213946	-8.9	-2.1	
	Bivariate(L/D)	Clayton	□ =0.325862473	-9.7	-2.9	
		Frank	□ =0.325862473	**-9.8**	**-3.0**	**BF**
		Gumbel	□ =1.24399673	-9.7	-2.9	
		Normal	ρ =0.4056	-2.9	-1.1	
		T	ν =1.3E+07, ρ =0.40998	-9.2	-2.4	
12 months	Bivariate(D/S)	Clayton	□ =3.967145742	NA	NA	
		Frank	□ =44.81371437	**18.68**	**7.29**	**BF**
		Gumbel	□ =10.44693912	NA	NA	
		Normal	ρ =0.979917481	16.68	NA	
		T	ν =1.00 ,ρ =0.9971	79.18	79.18	
	Bivariate(S/L)	Clayton	□ =0.9722	NA	NA	
		Frank	□ =4.1178	**16.68**	**7.29**	**BF**
		Gumbel	□ =2.0822	NA	NA	
		Normal	ρ =0.7989	NA	71.64	
		T	ν =4.7E+06, ρ =0.6653	NA	NA	
	Bivariate(L/D)	Clayton	□ =1.72	NA	NA	
		Frank	□ =4.905721079	**16.68**	**7.29**	**BF**
		Gumbel	□ =2.464838115	NA	NA	
		Normal	ρ =0.782095096	NA	69.52	
		T	ν =1.00, ρ =0.8393	79.18	69.8	

Bivariate modeling of drought using copula

Bivariate Archimedean (Clayton, Frank, Gumbel) and Metaelliptical (Normal and Student's t) copula families were chosen to model drought variables (severity, duration, and interval). For the drought variables duration and severity, Normal copula was found the best fit for SPI at 3 and 6 months scale and Frank copula was found the best fit for SPI at 12 months scale. For the drought variables severity and interval, Frank copula was found the best fit for SPI at 3, 6 and 12 months time scale. For the drought variables length and duration, Frank copula was found the best fit found the best fit for SPI at 3,6 and 12 months scale.(Table 7)

Return period analysis

The average expected drought interval time was found 25 months, 43.2 months and 68 months at 3, 6 and12 months time scales respectively. Therefore, these values were considered for calculation here after.

The value of drought severity, duration and interval at these return period at 3, 6 and 12 months time scales were calculated and presented in Table 8 and depicts a comparison of the return period obtained for univariate drought and associated multivariate primary ('or' and 'and' case) and secondary return periods accounting duration, severity and interval for bivariate cases.

For instance, at univariate return period of 100 years (3 months time scale), the values of duration, severity, and interval are calculated as 5.8, 3.37, 40.57 months. However, bivariate joint response of drought variables results in period of 12.1 years for T_{DS}^{and} (D ≥ 5.8 and S ≥ 3.37) and 7.4 years for T_{DS}^{or} (D ≥5.8 or S ≥ 3.37), whereas trivariate periods T_{DSL}^{and} (D ≥5.8 and S ≥3.37 and L ≥40.57) and T_{DSL}^{or} (i.e., D ≥5.8 and S ≥3.37 and L ≥40.57) were calculated as 1059.2 years and 7.4 years, respectively. It is observed from Table 8 that univariate analysis does not provide adequate information about risks associated with three drought variables.

Table 8: Comparison of univariate and bivariate return periods (in years) of drought characteristics

SPI time scale	T (years)	Duration (D) (months)	Severity (S)	Interval (L) (months)	T_{DS}^{or}	T_{SL}^{or}	T_{LD}^{or}	T_{DS}^{and}	T_{SL}^{and}	T_{ID}^{and}
3 months	2	1.2	0.4	0.66	1.8	1.8	1.8	2.8	2.9	2.9
	5	3.89	1.96	15.54	4.6	4.0	3.9	5.6	38.7	39.2
	10	5.8	3.37	40.57	8.6	8.4	8.3	12.1	513.3	551.4
	20	7.61	4.88	77.67	16.5	17.9	17.7	25.7	4375.7	4806.1
	50	10	7.12	150.95	38.8	46.3	45.8	68.1	55252.3	61574.0
	100	11.98	9.04	210	76.1	93.0	94.8	144.9	268062.1	309923.5
6 months	2	0.6	0.07	0.01	1.7	1.6	1.6	11.7	11.8	11.8
	5	0.8	0.11	0.06	4.8	4.8	4.6	11.9	12.1	12.0
	10	2.3	0.38	1.43	9.8	9.7	9.8	13.0	14.0	13.9
	20	9.6	2.15	39.5	19.5	15.2	15.2	20.5	29.2	28.9
	50	32.4	9.74	206.38	47.6	31.6	31.6	52.6	136.9	133.6
	100	59	20.59	220	93.9	41.5	41.5	106.6	268.9	260.7
12 months	2	0.77	0.5	0	1.6	1.5	1.5	24.8	12.6	24.6
	5	1.51	1.4	0.3	3.9	4.5	4.0	30.3	15.7	30.6
	10	24.3	1.6	1.87	9.7	7.4	4.9	35.0	17.9	35.9
	20	31.76	1.8	5.17	15.7	20.0	14.5	40.7	20.3	40.2
	50	99.44	2.4	143	20.1	43.5	20.8	86.5	49.9	99.0
	100	120	2.8	300	23.2	83.2	22.7	194.7	98.7	164.3

The analysis revealed that, in all the cases (Table 6), the return period obtained from univariate marginal distributions is greater than the joint return period in 'or' case (T^{or}) and less than the joint return period obtained in 'and' case (T^{and}). Hence, the occurrence of higher magnitude drought characteristics at a time in central Brahmaputra valley agro-climatic zone of Assam is less frequent.

The analysis deduces that univariate analysis alone is not enough to conclude drought risk related with three variables, drought risk will be overestimated if only 'or' case joint return period is taken into consideration, whereas the drought risk will be underestimated if only 'and' case joint return period are taken into consideration.

CONCLUSION

The meteorological drought was modelled using SPI 3, 6, 12 months time scales for central Brahmaputra valley agro-climatic zone of Assam.Univariate modelling of drought severity, duration and interval were done using continuous probability distribution functions. The joint dependence of drought properties- severity, duration, and interval-was modelled using Archimedean and Metaelliptical copulas. Univariate and joint return periods were also calculated. Important conclusions of the study are as follows:

1. At SPI 3 months time scale, moderately dry drought has the maximum frequency of occurrence followed by extremely dry and severely dry droughts. At 6 months time scale, extremely dry drought has the maximum frequency of occurrence followed by moderately dry and severely dry droughts. At 12 months time scale, extremely dry drought has the maximum frequencies of occurrence followed by severely and moderately dry droughts.
2. In all the cases, i.e., SPI-3, SPI-6 and SPI-12-time series, a significant falling trend was observed for the entire time period (1901-2013) and short-term trends time period (1979-2013)
3. In all the cases, i.e. SPI at 3, 6 and 12 months time scale, the return period obtained from univariate marginal distributions was found to be greater than the joint return period in 'or' case (T^{or}) and less than the joint return period obtained in 'and' case (T^{and})
4. The analysis deduces that univariate analysis alone is not enough to conclude drought risk related with three variables, drought risk will be overestimated if only 'or' case joint return period is taken into consideration, whereas the drought risk will be underestimated if only 'and' case joint return period are taken into consideration

The results obtained in this study can be used for water management planning of the study area.

REFERENCES

1. *Akaike, H. 1974. A new look at the statistical model identification. IEEE Trans Autom Control AC-19(6):716–723*

2. *Bonaccorso, B., Cancelliere, A., and Rossi, G. 2003. An analytical formulation of return period of drought severity. Stochastic Environmental Research and Risk Assessment, 17:157–174.*

3. *Bozdogan, H. 2000. Akaike's information criterion and recent developments in information complexity". J Math Psychol, 44:62–91*

4. *Cancelliere, A., and Salas, J. D. 2004. Drought length properties for periodic-stochastic hydrological data. Water Resour. Res., 40(2), W02503*

5. *Frahm G, Junker M, Schimdt, R.(2005). Estimating the tail dependence coefficient properties and pitfalls. Insurance: Mathematics and Economics, 37:80-100*

6. *González, J. and Valdés, J. B., 2003, 'Bivariate drought recurrence analysis using tree ring reconstructions', Journal of Hydrologic Engineering ASCE 8(5), 247–258.*

7. *Helsel, DR, Hirsch, R. M.(2002). Statistical methods in water resources; techniques of water resources investigation, USGS.*

8. *Jhajharia, D., Kumar, R., Dabral, P. P., Singh, V. P., Choudhary, R. R. and Dinpashoh 2014. Reference evapotranspiration under changing climate over the Thar Desert in India. Meteorological Applications, 22:425-435.*

9. *Joe, H.(1997). Multivariate model and dependence concepts, Chapman and Hall, London*

10. *Kendall, M. G.(1975). Rank correlation methods. 4th edition, Charles Griffin, London.*

11. *Kim, T.W. and Valdes, J.B. (2003) Nonlinear Model for Drought Forecasting Based on a Conjunction of Wavelet Transforms and Neural Networks. J. Hydrol. Eng., 8:319-328.*

12. *Mann, H. B.(1945). Non-parametric tests against trend, Econometrica, 33:245-259.*

13. *McKee , T.B., Doesken , N.J., Kliest, J. 1993. The relationship of drought frequency and duration time scales. Proceedings of the Eighth International Conference on Applied Climatology, Boston, MA: 179–184.*

14. *Mishra, A. K., and Desai , V. R. 2005. Spatial and temporal drought analysis in the Kansabati river basin, India , 3(1): 31-41.*

15. *Mishra, A.K., Singh V.P., Desai, V.R, 2009. Drought characterization: a probabilistic approach, Stoch. Environ. Res. Risk Assess. 23(10):41-55.*

16. *Mishra, A. K., and Singh, V. P.2010. A review of drought concepts. J. Hydrol., 391(1–2): 202–216.*

17. *Neath, A. and Cavanaugh, J.1997. Regression and time series model selection using variants of the Schwarz information criterion, Communication in Statistics, 26: 559-580.*

18. *Nelsen, R.B. 2006. An introduction to copulas, volume 139 of Lecture Notes in Statistics, Springer, New York.*

19. *Pandey, R.P., Dash, B.B., Mishra, S.K. and Singh, R..2008. Study of indices for drought characterization in KBK districts in Orissa(India). Hydrologic Process, 22: 1895-1907.*

20. *Pandey, R.P., Ramsastri, K.S.2001. Relationship between the common climatic parameter and average drought frequency. Hydrological Process, 15:1019-1032.*

21. *Sales, J.D.1986. State of the art of statistical techniques for describing drought characteristics. WARRDOO, International Seminar on drought analysis. Perugia, Italy.*

22. *Salas, J.D., Fu, C., Cancelliere, A., Dustin, D., Bode, D., Pineda, A., Vincent, E., (2005). Characterizing the severity and risk of drought in the Poudre River, Colorado. Journal of Water Resources Planning and Management 131:383–393.*

23. *Sen, P.K. 1968. Estimates of the regression coefficient based on Kendall's tau. J. Am. Stat. Assoc, 63:1379–1389.*

24. *Shiau, J. T., and Shen, H. W. 2001. Recurrence analysis of hydrologic droughts of differing severity. J. Water Resour. Plann. Manage,127(1), 30-40.*

25. *Shiau, J.T. 2003. Return period of bivariate distributed hydrological events. Stochastic Environmental Research and Risk Assessment, 17(1-2): 42-57.*

26. *Shiau, J. 2006. Fitting drought duration and severity with two-dimensional copulas. Water Resour. Manage., 20(5) 795-815.*

27. *Shrivastava, S. K., Rai, R. K., and Pandey, A.2008. Assessment of meteorological droughts in North Lakhimpur district of Assam, J. Indian Water Resor. Society, 28(2) : 26-31.*

28. *Sklar, K.1959. Functions de repartiton Dimensions et Leura Marges, Publ. Inst. Stat. Univ. Paris, 8:229-231.*

29. *Song, S.B., Singh, V.P., 2010a. Frequency analysis of droughts using the Plackett copula and parameter estimation by genetic algorithm. Stoch. Environ. Res. Risk Assess., 24, 783–805.*

30. *Song, S.B., Singh, V.P., 2010b. Meta-elliptical copulas for drought frequency analysis of periodic hydrologic data. Stoch. Environ. Res. Risk Assess., 24, 425–444*

31. *Tiwari, K. N., Paul, D.K., Gontia, N K.(2007). Characterization of meteorological drought. Hydrology Journal(IAH), 30(12-2) :15-27.*

32. *Thiel, H.(1950).A rank-invariant method of 1inear and polynomial regression analysis. Part 3, Proceedings of Koninalijke Nederlandse Akademie van Weinenschatpen A., 53: 1397-1412*

33. *Thornthwaite, C.W. (1948). An approach towards a rational classification of climate, Geographical Review, 38(1):55-94.*