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OPTIMAL CANAL SECTION WITH MINIMISING LINING MATERIAL COST

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ABSTRACT

A canal is a hydraulic structure, which is provided to convey water from a source to a required point. Lining is a coating provided in the surface of canal section to prevent seepage loss and to provide stability to the section. The lining may be uniform lining, composite lining or partial lining. When the same lining material with uniform cost is provided over the cross section of the canal then it is termed as uniform lining. When different lining material with non uniform cost is provided over the cross section then it is known as composite lining or non uniform lining. When lining is provided either the base or the sides then it is termed a partial lining. In practice generally composite lining is provided over the cross section. In this study, an attempt has been made to design an optimal canal section with consideration of minimum cost of lining material. Free board is also considered in the design which is considered as discharge dependent as well as depth dependent.

Keywords: *Uniform Lining, Composite Lining, Partial Lining, Optimal Canal Section*

INTRODUCTION

A canal section is provided to convey water from a source to the required point. As per the field conditions available, lining can be provided in following modes

i) Canal section with uniform lining
ii) Canal section with composite lining
iii) Canal section with partial lining

A canal section can be designed by the following approaches depending upon the data available for canal designed

1) Flow area approach- In this approach velocity of the flow through the canal section is specified. The design discharge is known and from the specified velocity, flow area can be calculated for which the canal section is designed.

2) Section factor approach- The canal section is designed for the specified section factor ($AR^{2/3}$) which is also function of design discharge, roughness and bed slope of the canal section.

These approaches are used to design the lined canal section. Optimal canal section may be designed by three considerations.

a) By minimizing the total area of construction.
b) By minimizing the cost of lining material.
c) By minimizing the total cost of construction.

The first consideration may be termed as shape optimization as total area of section is to be optimized. The second consideration may be termed as material optimization as the cost of lining material is to be optimized and third one is considered as total optimization as the total cost of construction is to be optimized. The free board is also as essential parameter in the canal design of a canal section which is also considered in optimal design of a canal section. Which can be considered as discharge dependent as specified by Indian standards and depth dependent as recommended by USBR. In this paper an optimal section is obtained by minimizing the cost of lining materials provided with composite lining. Free board is also considered in the canal design.

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PARAMETRIC RELATIONSHIPS-

Geometric Parametric –The flow area A and wetted perimeter ‘P’ of a trapezoidal section as shown in Fig 1 are expressed as

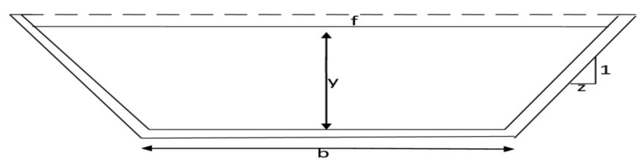

Fig. 1

$$A = by + zy^2$$

And $$P = b + 2y\sqrt{1+z^2}$$

The cost of lining C for the section per unit length of the canal is given by

$$C = b\,C_b + 2\,(y+f)\sqrt{1+z^2}\,C_s$$

Where, b = base width , m

y = depth of flow, m

z = side slope 1V : zH

f = free board, m

C_b = cost of lining material in Rs per sq.m for the base

C_s = Cost lining material in Rs per sq.m for the sides

Flow Equations –

For the flow computation, manning’s equation is used and the section factor can be expressed as

$$Z = AR^{2/3} = \frac{Qne}{\sqrt{S_0}}$$

Where, R = hydraulic mean depth, Q = discharge

n_e = Manning's reposite coefficient , S_o = bed slope of the canal

if Manning's rugocity coefficient for base and sides is taken as different the equivalent roughness can be calculated as

$$n_e = \left[\frac{2n_s^{1.5}\, y\sqrt{1+z^2}+n_b^{1.5}\, b}{b+2y\sqrt{1+z^2}}\right]^{2/3}$$

Using equation 1,2 and 4 depths of flow is computed as

$$y = \frac{\left[\frac{b}{y}+2\sqrt{1+z^2}\right]^{1/4}}{\left[\frac{b}{y}+z\right]^{5/8}}\left[\frac{Qn_e}{\sqrt{S_0}}\right]^{3/8}$$

Or $y = \Phi\left[\frac{Qn_e}{\sqrt{S_0}}\right]^{3/8}$

Where, $\Phi = f\left(\frac{b}{y}, z\right)$

OPTIMISTIC APPROACH-

(1) Flow area approach- velocity of flow specified case for the given discharge as specified velocity of flow area A can be given by

$$A = \frac{Q}{V}$$

Case 1 – Discharge dependent free board

$$f = f(Q)$$

as discharge is constant free board f is also treated as constant.

The cost of lining per unit length is given by equation 3

$$C = b\,C_b + 2\,(y + f)\sqrt{1+z^2}\,C_s$$

For the minimum cost of lining differentiated cost C w.r.to depth of flow y is equated to zero

$$\frac{dC}{dy} = 0$$

Solving above equations, we get

$$\frac{b}{y} = 2\left(\sqrt{1+z^2}\,\frac{Cs}{Cb} - z\right)$$

The flow area $A = \frac{Q}{V}$

$by + zy^2 = \frac{Q}{V}$

$y^2\left[\frac{b}{y}+z\right] = \frac{Q}{V}$

$y = \frac{1}{\sqrt{\frac{b}{y}+z}}\sqrt{\frac{Q}{V}}$

and $b = \frac{b}{y}\,y$

value of b and y can be determined

Case II- Depth dependent free board:-

free board of can be expressed as

$f = k\sqrt{y}$

where k is a factor varies from 0.6 to 0.7

The total cost of lining from equation 3

$$C = b\,C_b + 2\,(y + f)\sqrt{1+z^2}\,C_s$$

For the minimize cost, $\frac{dC}{dy} = 0$

And we will get

$$\frac{b}{y} = M + \frac{m}{\sqrt{y}}$$

Where $M = 2\,\frac{C_s}{C_b}\sqrt{1+z^2} - 2z$

$m = \frac{C_s}{C_b}\,k\sqrt{1+z^2}$

For a given flow area

$A = \frac{Q}{V}$

$by + zy^2 = \frac{Q}{V}$

$\frac{b}{y} = \frac{Q}{Vy^2} - z$

Equating equation

$$M + \frac{m}{\sqrt{y}} = \frac{Q}{Vy^2} - z$$

Value of M,m,Q,V and z are known ,the value of depth y can be calculated by trial and error .

The value of $\frac{b}{y}$ is calculated for equation and b is calculated by

$b = \left(\frac{b}{y}\right)\,y$

Illustrative Example 1: Design the optimal canal section by using flow area approaches for the following data
$Q = 60\ m^3/s$, $V = 1.2$ m/s , $z = 1.5$, C_s = Rs 300/sq.m, C_b= Rs 200/sq.m

Discharge dependent free board:

Assuming f = 0.8m

Using equation

$$\frac{b}{y} = 2\left[\sqrt{1+z^2}\,\frac{C_s}{C_b} - z\right]$$

$$= 2\left[\sqrt{1+1.5^2}\,\frac{300}{200} - 1.5\right] = 2.4$$

Depth of flow

$$y = \frac{1}{\sqrt{\frac{b}{y}+z}}\sqrt{\frac{Q}{V}}$$

$$= \frac{1}{\sqrt{2.4+1.5}}\sqrt{\frac{60}{1.2}} = 3.58\text{m}$$

Base width $b = \frac{b}{y}\,y = 2.4 \times 3.58 = 8.59$m

Cost of lining per meter length,

$$C = bC_b + 2\,(y+f)\sqrt{1+z^2}\,C_s$$

$$= 8.59 \times 200 + 2\,(3.58+0.8)\sqrt{1+1.5^2} \times 300$$

C = Rs 6455.69

Check, Let us assume $\frac{b}{y} = 2.5 > 2.4$

Calculate values of b = 8.89, y = 3.54

Cost per 'm' length C = Rs 6464.43

Let $\frac{b}{y}$ = 2.3 < 2.4

Calculated value of b = 8.35m, y = 3.69m

Cost per 'm' length C = Rs 6461.77, hence 6461.77>6455.69 and 6464.43>6455.65

Hence value of cost for $\frac{b}{y}$= 2.4 is minimum

Depth dependent free board:

Assuming k = 0.7

Using equation$\frac{b}{y}$,

$$\frac{b}{y} = M + \frac{m}{\sqrt{y}}$$

Value of M = 2 [$\sqrt{1+z^2}\,\frac{C_s}{C_b}$ - 2z]

$$= 2\,[\sqrt{1+1.5^2}\,\frac{300}{200} - 2 \times 1.5\,] = 2.408$$

Value of m = $\frac{C_s}{C_b}$ k $\sqrt{1+z^2} = \frac{300}{200}$ x 0.7 $\sqrt{1+1.5^2}$ = 1.893

Using equation

$$M + \frac{m}{\sqrt{y}} = \frac{Q}{Vy^2} - z$$

$$2.408 + \frac{1.893}{\sqrt{y}} = \frac{60}{1.2y^2} - 1.5$$

Solving for y by trial and error

y = 3.171 m

$$\frac{b}{y} = 2.408 + \frac{1.893}{\sqrt{3.417}} = 3.471$$

b = 11.006m

free board f = 0.7$\sqrt{3.171}$ = 1.246m

Cost of lining per m length

C = 11.006 x 200 + 2(3.171+1.246) $\sqrt{1+1.5^2}$ x 300

= 2201.2 + 4777.71 = Rs 6978.92

Check, Assuming $\frac{b}{y}$= 3.6>3.471

$$Y = \frac{1}{\sqrt{\frac{b}{y}+z}}\sqrt{\frac{Q}{V}} = \frac{1}{\sqrt{3.6+1.5}}\sqrt{\frac{60}{1.2}} = 3.131\text{m}$$

b = 3.6 x 3.131 = 11.272 m

f = 0.7 x $\sqrt{3.131}$ = 1.238

Cost of lining per 'm' length

C = 11.272 x 200 + 2(3.131+1.238) $\sqrt{1+1.5^2}$ x 300

= 2254.4+ 4725.8 = Rs.6980.2

Assuming $\frac{b}{y}$ = 3.4<3.471

$$y = \frac{1}{\sqrt{3.4+1.5}}\sqrt{\frac{60}{1.2}} = 3.194\text{m}$$

b = 3.4 x 3.194 = 10.861 m

f = 0.7 $\sqrt{3.194}$ = 1.251

cost per meter length ,

C = 10.861 x 200 + 2 (3.194+1.251) $\sqrt{1+1.5^2}$ x 300

= 2172.2 + 4808 = Rs 6980.2 hence 6980.2>6978.92 and 6980.2

Hence section with b = 11.006m, y = 3.171m and cost Rs 6978.92 is optimal.

Section factor approach:

The optimal canal section is obtained for a given value of section factor

$Z = A\,R^{2/3}$

By Manning's equation discharge Q is given by

$$Q = \frac{A}{n_e} R^{2/3} S^{1/2} \text{ or } \frac{Q}{\sqrt{S}} = \frac{A}{n_e} R^{2/3} = K$$

Where K = conveyance factor

Equivalent roughness n_e can be expressed as

$$n_e = \left[\frac{2n_s^{1.5} y\sqrt{1+z^2} + n_b^{1.5} b}{b+2y\sqrt{1+z^2}}\right]^{2/3}$$

where n_s = roughness of side lining material

n_b = roughness of base lining material

In this approach, the optimal section is determined by using Lagrangian undetermined multiplier technique According to this technique the marginal changes in cost should be equal to marginal change in conveyance factor.

Which can be expressed as?

$$\frac{\partial K/\partial b}{\partial K/\partial y} = \frac{\partial C/\partial b}{\partial C/\partial y}$$

$$K = \frac{A}{n_e} R^{2/3} = \frac{A^{5/3}}{n_e P^{2/3}} A^{2/3}$$

$$= \frac{A^{5/3}}{n_e P^{2/3}} = \frac{[by+zy^2]^{5/3}}{[2n_s^{1.5} y\sqrt{1+z^2} + n_b^{1.5} b]^{2/3}}$$

$$\frac{\partial K/\partial b}{\partial K/\partial y} = \frac{3\left(\frac{b}{y}\right) + 10n_r^{1.5}\sqrt{1+z^2} - 2z}{5\left(\frac{b}{y}\right)^2 + 10\left(\frac{b}{y}\right)z + n_s^{1.5} y\sqrt{1+z^2}\left[6\left(\frac{b}{y}\right) + 16z\right]}$$

Case I- Discharge dependent free board

f= f(Q) , As Q = Constant, Also f = constant

Using Equation (3) and solving for $\frac{\partial C/\partial b}{\partial C/\partial y}$, we get

$$\frac{\partial C/\partial b}{\partial C/\partial y} = \frac{C_b}{2\sqrt{1+z^2}C_s}$$

Equating Equation (17) and (18), and solving for $\frac{b}{y}$, rearranged as

$$\frac{b}{y} = \frac{K_1 + \sqrt{K_1^2 + 20K_2}}{10}$$

where $K_1 = 6\sqrt{1+z^2}\left(\frac{C_s}{C_b} - n_r^{1.5}\right) - 10z$

$$K_2 = 20n_r^{1.5}\left(\frac{C_s}{C_b}\right)(1+z^2) - 4z\sqrt{1+z^2}\left[\frac{C_s}{C_b} + 4n_r^{1.5}\right]$$

We know that,

$\frac{Q}{\sqrt{S}} = \frac{AR^{2/3}}{n_e}$

Substitute value of A and R interms of b and y and solving for y,

$y = \frac{[2n_s^{1.5}y\sqrt{1+z^2}+b/7)]1/4}{\left[\frac{b}{7}+z\right]5/8}\left(\frac{Qn_b}{\sqrt{S}}\right)^{3/8}$

And b can be determined as

$b = \left(\frac{b}{y}\right) y$

Case-II Depth dependent free board:

free board $f = m\sqrt{y}$ here $m = \frac{f}{\sqrt{y}}$

Using Equation (3) and solving for $\frac{\partial C/\partial b}{\partial C/\partial y} = \frac{C_b}{\sqrt{1+z^2}C_s(z+m)}$

Using Equation (17) and (23) and solving for $\frac{b}{y}$ rearranged as

$\frac{b}{y} = \frac{K_3+\sqrt{K_3^2+20K_4}}{10}$

where $K_3 = 6\sqrt{1+z^2}\left[\frac{C_s}{C_b} + \frac{m}{2}\frac{C_s}{C_b} - n_r^{1.5}\right]$-10z

$K_4 = 20n_r^{1.5}\left(\frac{C_s}{C_b}\right)(1+Z^2)\left(1+\frac{m}{2}\right) - 4z\sqrt{1+z^2}\left(\frac{C_s}{C_b} + \frac{m}{2}\frac{C_s}{C_b} - 4n_r^{1.5}\right)$

where , $n_r = \frac{n_s}{n_b}$ $m = \frac{f}{\sqrt{y}}$

But we know that

$\frac{Q}{\sqrt{S}} = \frac{AR^{2/3}}{n_e}$

Substituting values of A and R in terms of b and y and solving for y

$y = \frac{[(2n_s^{1.5}y\sqrt{1+z^2}+b/7)]^{1/4}}{[\frac{b}{y}+z]^{5/8}}\left(\frac{Qn_b}{\sqrt{S}}\right)^{3/8}$

And b can be determined as, $b = \left(\frac{b}{y}\right) y$

Illustration Example -2

The problem described in illustrative example –I is to be designed by section factor approach. Additional data are k = 0.7, roughness of side material $n_s = 0.015$, Roughness of base material $n_b = 0.017$, Bed slope of the canal S = 1 in 4000

Case – I Discharge dependent free board-

Roughness ratio, $n_r = \frac{n_s}{n_b} = 0.015/0.017 = 0.8823$

Factor $K_1 = 6\sqrt{1+z^2}\left(\frac{C_s}{C_b} - n_r^{1.5}\right) - 10z = 6\sqrt{1+1.5^2}\left(\frac{300}{200} - 0.8823^{1.5}\right) - 10 \times 1.5 = -7.735$

$K_2 = 20n_r^{1.5}\left(\frac{C_s}{C_b}\right)(1+z^2) - 4z\sqrt{1+z^2}\left[\frac{C_s}{C_b}+4n_r^{1.5}\right] = 28.702$

$\frac{b}{y} = \frac{K_1+\sqrt{K_1^2+20K_2}}{10} = \frac{-7.735+\sqrt{-7.735^2+20x28.702}}{10} = 1.744$

Depth of flow

$y = \frac{[(2n_r^{1.5}y\sqrt{1+z^2}+b/7)]^{1/4}}{[\frac{b}{y}+z]^{5/8}}\left(\frac{Qn_b}{\sqrt{S}}\right)^{3/8} = 3.37$ m

Base width $b = \left(\frac{b}{y}\right) y = 1.744 \times 3.37 = 5.877$m

Cost of lining per 'm' length

$C = b\,C_b + 2\,(y+f)\sqrt{1+z^2}\,C_s$ = Rs 5687.95

(II) Depth dependent free board

$f = k\sqrt{y}$, k = 0.7

$m = \frac{f}{\sqrt{y}} = 0.7$, $n_r = 0.8823$

Factor $K_3 = 6\sqrt{1+z^2}\left(\frac{C_s}{C_b} + \frac{m}{2}\frac{C_s}{C_b} - n_r^{1.5}\right) - 10z = -2.056$

$K_4 = 20n_r^{1.5}\left(\frac{C_s}{C_b}\right)(1+Z^2)\left(1+\frac{m}{2}\right) - 4z\sqrt{1+z^2}\left(\frac{C_s}{C_b} + \frac{m}{2}\frac{C_s}{C_b} - 4_r^{1.5}\right)$

= 51.285

$\frac{b}{y} = \frac{K_3+\sqrt{K_3^2+20K_4}}{10} = 3.004$

y = 2.914m, b = 8.751 m , f = 1.195 m

cost of lining per 'm' length , C = Rs 6193.68

CONCLUSION

The paper deals with the design of an optimal canal section corresponding to the minimum cost of lining materials, which is named as here optimization. Two approaches are proposed here. Flow area approach is quite simple and wider section of the canal. Section factor approach is quite difficult and gives narrower section, but this section is more optimal than that of obtained by flow area approach.

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