



COMPARING SPATIAL INTERPOLATION METHODS FOR CMIP5 MONTHLY PRECIPITATION AT CATCHMENT SCALE

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ABSTRACT

Use of Regional Climate Models (RCMs) is prevalent in downscaling the large scale climate information from the General Circulation Models (GCMs) to local scale. But it is computationally intensive and requires application of a numerical weather prediction model. For more straightforward computation, spatial interpolation are commonly used to re-gridding the GCM data to local scales. There are many interpolation methods available, but mostly they are chosen randomly, especially for GCM data. This study compared eight interpolation methods (linear, bi-linear, nearest neighbour, distance weighted average, inverse distance weighted average, first-order conservative, second-order conservative and bi-cubic interpolation) for re-gridding of CMIP5 decadal experimental data to a catchment scale. For this, CMIP5 decadal precipitation data from three GCMs were collected and subset for Australia and then re-gridded to 0.05 degree spatial resolution matching with the observed gridded data. The re-gridded data were subset for Brisbane catchment in Queensland, Australia and a number of skill tests (root mean squared error, mean absolute error, correlation coefficient, Pearson correlation, Kendal's tau correlation and index of agreement) were conducted for a selected observed point to check the performances of different interpolation methods. Additionally, temporal skills were computed over the entire catchment and compared. Based on the skill tests over the study area, the second-order conservative (SOC) method was found to be an appropriate choice for interpolating the gridded dataset.

Keywords: Comparison, Interpolation, Precipitation, Spatial and Catchment

INTRODUCTION

General Circulation Models (GCMs) are widely used to assess climate change and its potential impacts at different temporal and spatial scales, but their coarse spatial resolution (100-250 km) is inadequate for their application at a local scale due to lack of spatial details [1]–[3]. The Regional Climate Models (RCMs) are often used to downscale the large scale climate information from GCMs to a local scale; however, RCMs are complicated, computationally intensive and time-consuming. To avoid this complexity, in practice, spatial interpolations are applied [4]–[7] to re-grid the coarser resolution climate model data onto a finer resolution. However, in most cases, spatial interpolation methods are randomly used. For instance, bilinear interpolation has been used in many studies [8]–[10] but the reason behind selecting the bilinear method was not well explained. Climate variables such as precipitation shows high spatial variability in frequency and magnitude, where, understanding the spatial distribution of precipitation at different spatial scales is important for water resource management, hydrological modelling, agricultural industries and urban planning. Therefore, the selection of an appropriate spatial interpolation method is important to provide the accurate spatial distribution of the precipitation when transforming from a relatively coarser to a finer spatial resolution.

Various spatial interpolation techniques ranging from

simple to complex have already been used for remapping data to a desired finer resolution [5], [11]. For interpolating the rain gauge station data at small and medium scale catchments (or basins), Nearest Neighbour (NN), Inverse Distance Weighting (IDW), Thiessen polygons, Spline and different forms of Kriging are frequently used [12]. Many studies have compared the performance of these spatial interpolation methods for the rainfall data at different temporal and spatial scale. For instance, da Silva et al. [13] compared seven interpolation methods for the monthly precipitation over Pernambuco, Brazil and reported trend surface analysis to be the best followed by natural NN, IDW and Kriging. Yang et al. [4] compared four methods with the model generated daily precipitation data and reported that IDW performed slightly better than Spline, Kriging and ANUDEM [14]. Dirks et al. [15] didn't find any advantage of using Kriging over IDW, Thiessen or areal-mean while gridding rainfall data from 13 rain gauge stations on Norfolk Island. Consequently, Wu et al. [16] evaluated a number of spatial interpolation methods for mapping the bathymetry of lowermost Mississippi River, which includes IDW, Ordinary Kriging (OK), Universal Kriging (UK), Radial Basis Function (RBF), local Polynomial and anisotropic form of Elliptical IDW, and OK and found that both the RBF and anisotropic form of OK performed best. Zhang et al. [17], compared OK, co-Kriging with elevation as covariate (Cok-elevation) and co-Kriging with precipitation data from tropical rainfall measuring mission to interpolate precipitation data from 39 rain gauge stations in the Tibetan Plateau and reported that Cok-TRMM is more effective than the other two, which was also confirmed by Foehn et al. [18]. Note, the performance of the interpolation methods depends on several factors, in

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particular, temporal and spatial resolution of the considered data and the study region. Degré et al.[19] reviewed a number of spatial interpolation methods from different perspective and concluded that, for annual and monthly rainfall, geo-statistical interpolation methods (different mood of Kriging) seem preferable to the deterministic methods (Thiessen, NN, IDW etc.), but for the daily rainfall, geo-statistical methods and IDW can be a better option. Most of the aforementioned studies interpolated the rain gauge station data and evaluated the interpolation methods at selected points within the study areaby using error metrics; root mean squared error (RMSE), mean absolute error, mean standard errors. To get an idea of which method produces better interpolation at the catchment level, it is essential to apply the methods for the entire study area in addition to a single point. Wagner et al. [20]also suggested that evaluation of the interpolation methods should include the spatial distribution over the study area for precipitation data. Therefore the objective of this study is to evaluate the different interpolation methods for the application of GCM data in a catchment level. This study will consider a single point measure as well as entire catchment for spatial distribution of precipitation data. In addition, this study also assesses the performance of interpolation methods due to the change in spatial resolution of the selected data sets.

MATERIALS AND METHODS

Data Collection

CMIP5 experiments (e.g., decadal) provide global climate data for a wide range of climate variables generated from a number of climate models. The decadal simulation, once initialized, generate climate data for ten years and longer in some cases[21]. Monthly precipitation data from three GCMs; MIROC4h, EC-EARTH and MPI-ESM-LR was downloaded from the CMIP5 data portal (<https://esgf-node.llnl.gov/projects/cmip5/>). Details of the models and the data are given in Table 1.

The gridded monthly precipitation data with a spatial resolution of 0.05 degree was collected from the Australian Bureau of Meteorology (BoM). The gridded observed data of BoM were produced by the Australian Water Availability

Project (AWAP)[22].

Data processing

Firstly, the model datasets were subset for the Australian region, thereafter, all the available ensembles (i.e., multiple runs of the same model with slightly perturbed initial conditions) of the individual models are averaged to produce a single dataset for each model. These datasets were then interpolated from their native grids onto 0.05 x 0.05 degree matching with the grid of the observed dataset. Finally, the interpolated data were subset for the selected Brisbane catchment (i.e., longitude from 151.70 E to 153.150 E and latitude from 26.50 S to 28.150 S) in Queensland, Australia.

Interpolation methods

In this study, eight different interpolation methods were evaluated. The six methods; Bi-linear (BiLIN), Nearest Neighbor (NN), Distance Weighted Average (DWA), First-order conservative (FOC), Second Order conservative (SOC) and Bi-Cubic (BIC) interpolation were performed by the Climate Data Operator (CDO)[23] tool, whilst Linear (LIN) and Inverse-Distance Weighted Average (IDW) were performed by the Matplotlib and Scipy libraries in Python. It is worth noting that DWA is also an IDW method, where four nearest neighbour points (by default) are used, whilst the Scipy based IDW method, only three nearest neighbour points are considered.

Linear interpolation is the concatenation of linear interpolants between each pair of data points. But the “LinearTriInterpolator” from Matplotlib performs linear interpolation on a triangular grid. Each triangle is represented by a plane so that interpolated values lie on that plane of the triangle containing the interpolants. For the Inverse-Distance Weighted Average, Scipy spatial algorithm described by Maneewongvatana and Mount [24] is used to locate the neighbouring points for a given set of points.

CDO uses adapted interpolation methods from the SCRIP library [25]. SCRIP is a software package.It computes the addresses and weights for remapping and interpolating variables between grids on the spherical coordinates. Initially,it was written for remapping the fields to desired

Table 1: Model used in this study

Modelling Centre (or Group)	Model Name	(Atmospheric Resolutions in degree)	Time span
Atmosphere and Ocean Research Institute (The University of Tokyo), National Institute for Environmental Studies, and Japan Agency for Marine-Earth Science and Technology	MIROC4h	(0.5625 X 0.5616)	10 Year; From January 1991 to December 2000
Meteorological Research Institute	EC-EARTH	(1.125 X 1.1215)	
Max Planck Institute for Meteorology	MPI-ESM-LR	(1.875 X 1.865)	

grids in a coupled climate model but can also be used for other applications.

Performance Assessment

The observed dataset has 496 grid (5.0 km X 5.0 km) points within the Brisbane catchment, and the skill tests are performed at the grid point (latitude 27.50 S and longitude 153.050 E) located closest to an AWS (Automated weather stations) rain gauge (the observed point at latitude 27.480 S and longitude 153.040 E) operated by the Bureau of Meteorology, Australia. To assess the performance, five skill tests: root mean squared error (RMSE), mean absolute error (MAE), correlation coefficient (CC), anomaly correlation coefficient (ACC) according to Wilks (2011) [26] and index of agreement (IA) suggested by Wilmot [27] were used.

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (F_i - O_i)^2}, \quad MAE = \frac{1}{N} \sum_{i=1}^N |F_i - O_i|$$

$$CC = \frac{\sum(F - \bar{F})(O - \bar{O})}{\sqrt{\sum(F - \bar{F})^2} \sqrt{\sum(O - \bar{O})^2}}; \quad ACC = \frac{\sum\{(F - C) - (\bar{F} - \bar{C})\} * \{(O - C) - (\bar{O} - \bar{C})\}}{\sqrt{\sum(F - C)^2} \sqrt{\sum(O - C)^2}}$$

$$IA = 1 - \frac{\sum_{i=1}^n (F_i - O_i)^2}{\sum_{i=1}^n (|F_i - \bar{O}| + |O_i - \bar{O}|)^2}$$

F and *O* present modelled (interpolated) and observed values respectively whilst \bar{F} , \bar{O} present their annual mean, and *C* is the decadal mean of the observed (BoM) data.

Additionally, Pearson correlation (Pr) and Kendall's tau (Kr) correlations are also calculated and compared.

RESULTS AND ANALYSIS

The monthly precipitation data from three CMIP5 models are evaluated against the observed data, and the results are presented in Table 2. These models with three different spatial resolutions were chosen (see Table 1) to assess the effect on skills of the interpolation methods due to the variations of atmospheric spatial resolution (before interpolation) of the interpolant dataset. The results for the interpolation methods aren't significantly different, but to some extent, some of them are slightly better than others.

Overall, DWA method has comparatively lower errors for all three selected models with varying values for the skill tests; CC, ACC, IA, Pr and Kr. However, the performance of the interpolation methods is sensitive to the choice of models; and the spatial resolution of the interpolant dataset. DWA has the lowest errors (RMSE and MAE) and IA, whereas LIN has the highest values for CC, ACC, Pr followed by DWA (see Table 2), whilst SOC and FOC performed poorly on all skill tests except IA. Overall, the DWA method has the lowest errors for all three models and outperforms all methods on all temporal skill for the MIROC4h model at a single grid point. With the change in spatial resolutions, the skill specifically CC, ACC, Pr, Kr and IA varied a little with little to no change in RMSE and MAE. Note, Table 2, presents temporal skills at the observed station only.

These study also compared the spatial variations of these temporal skills over the entire catchment for all three models, but only IA (Fig. 1) and RMSE (Fig. 2) for the MIROC4h models are presented here. From the spatial comparison, it is evident that NN along with conservative methods found little better in CC, ACC (not shown) and RMSE whilst DWA outperforms other methods for IA. An overview of spatial comparison of all three models based on

Table 2: Comparison of Interpolation Methods Based on Different Temporal Skills of Selected Models against Observed Monthly Precipitation at a single Gridpoint. Higher (Lower) the Skill (RMSE) Values will Present Higher the Performance of Interpolation Methods. For the sake of brevity, Pr and Kr are not presented here.

Interp.	MIROC4h				EC-EARTH				MPI-ESM-LR			
	RMSE	CC	ACC	IA	RMSE	CC	ACC	IA	RMSE	CC	ACC	IA
BiLIN	80.991	0.368	0.354	0.539	79.377	0.437	0.362	0.458	77.432	0.338	0.306	0.414
LIN	80.948	0.368	0.355	0.539	79.377	0.437	0.362	0.458	77.407	0.343	0.307	0.414
NN	82.634	0.345	0.334	0.520	79.207	0.438	0.364	0.462	80.083	0.325	0.289	0.437
IDW	80.998	0.366	0.353	0.536	79.221	0.438	0.364	0.461	77.994	0.334	0.301	0.423
DWA	79.744	0.380	0.365	0.540	79.060	0.436	0.363	0.458	77.284	0.339	0.306	0.405
FOC	82.303	0.350	0.338	0.525	79.204	0.438	0.364	0.462	80.100	0.325	0.289	0.437
SOC	82.307	0.350	0.338	0.525	79.207	0.438	0.364	0.462	80.083	0.325	0.289	0.436
BIC	81.414	0.362	0.349	0.536	79.480	0.438	0.363	0.458	78.085	0.332	0.300	0.424

the specified thresholds of individual skills is presented in Table 3. From this comparison, it is evident that NN, DWA and the conservative methods perform better than others with little variations in skills over model types where SOC found more consistent, followed by FOC in better performance than DWA and NN.

DISCUSSION AND CONCLUSION

This study compared different spatial interpolation methods at a catchment scale, where the temporal errors and skills were evaluated at an observed point within the catchment and spatial comparison of temporal skills for the whole catchment. Preliminary results show no significant

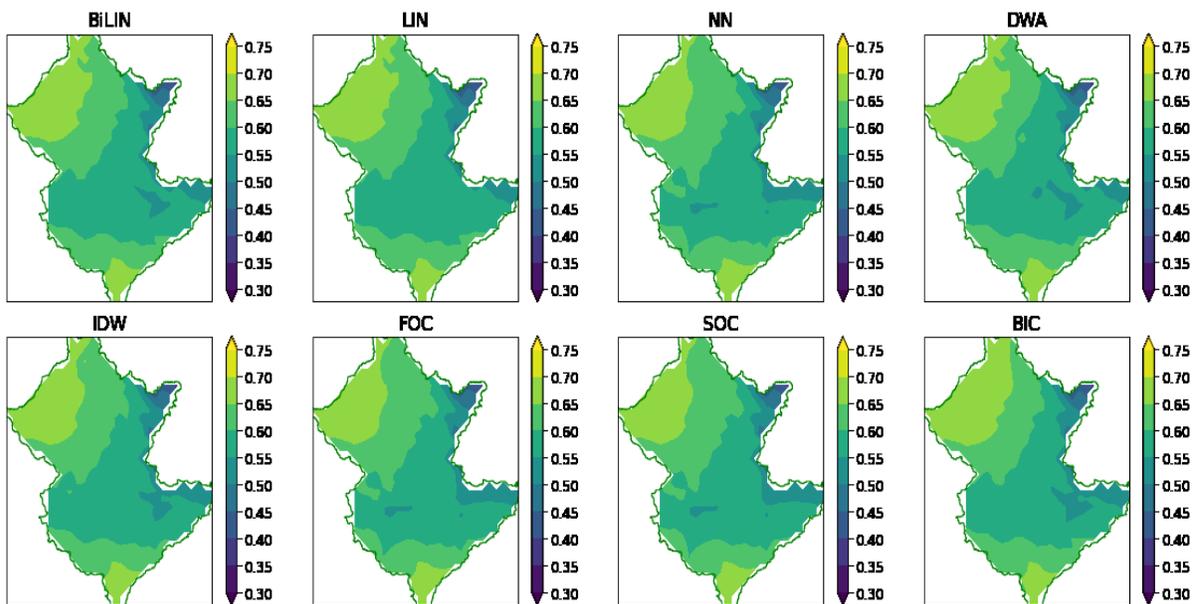


Fig.1: Spatial comparison of Index of Agreement (IA) of different interpolation methods (MIROC4h) over the catchment. Labels on the right of each plot indicate more the brightest area higher the performance of the interpolation methods.

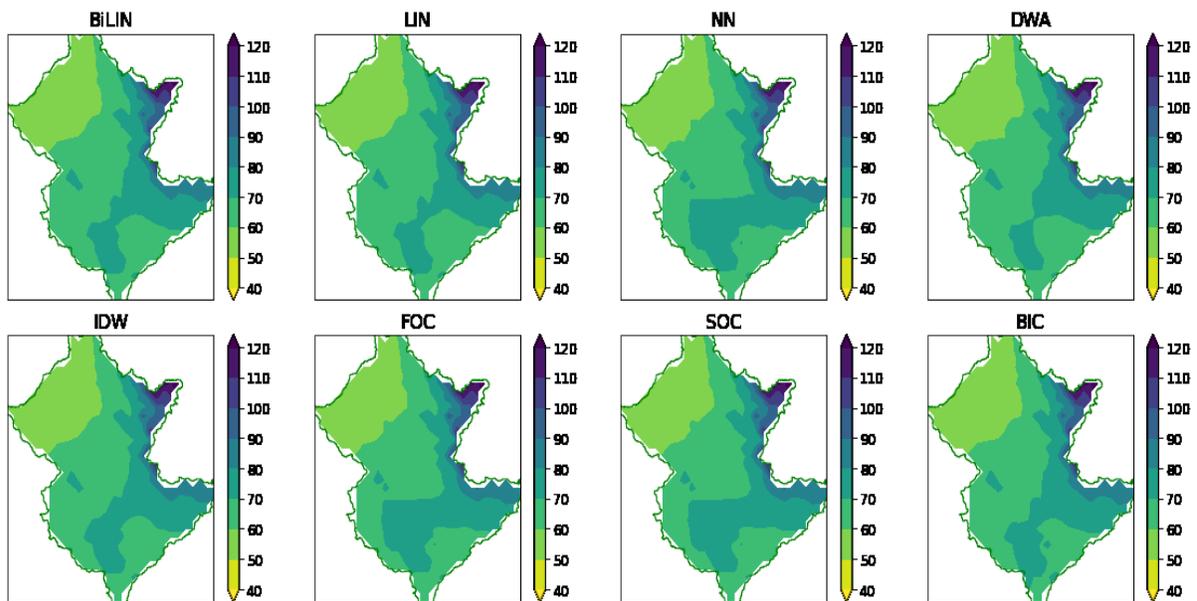


Fig. 2: Comparison of the spatial variations of Root Mean Squared Error (RMSE) of different interpolation methods (MIROC4h) over the catchment. Labels on the right of each plot indicate more the brightest area higher the performance of the interpolation methods.

Table 3: Number of Grids Covered by the Interpolation Methods for the Specific Thresholds of the Skills. Selected Models, Skills and corresponding thresholds are presented in the first, second and third row respectively. Higher the number in the respective columns presents better the performance of the interpolation methods over the catchment and vice versa.

Interp.	MIROC4h				EC-EARTH				MPI-ESM-LR			
	<i>RMSE</i>	<i>CC</i>	<i>ACC</i>	<i>IA</i>	<i>RMSE</i>	<i>CC</i>	<i>ACC</i>	<i>IA</i>	<i>RMSE</i>	<i>CC</i>	<i>ACC</i>	<i>IA</i>
	<=55	>=0.55	>=0.5	>=0.65	<=55	>=0.55	>=0.45	>=0.6	<=55	>=0.45	>=0.35	>=0.6
BiLIN	47	5	11	102	22	9	0	5	20	3	8	87
LIN	45	9	16	102	22	11	0	5	20	2	2	84
NN	52	14	20	88	56	0	3	22	19	1	16	85
IDW	50	6	12	92	19	0	0	3	19	3	2	79
DWA	45	2	8	103	20	11	1	6	19	11	27	86
FOC	52	14	21	88	56	0	3	21	19	1	15	85
SOC	52	14	21	88	58	0	3	22	19	1	14	87
BIC	51	11	19	112	23	12	0	5	17	3	5	97

difference among the interpolation methods when compared at observed stations. This may be because of the interpolant datasets is regularly gridded as opposed to the irregularly distributed point rain gauge stations. For irregular datasets such as point rain gauge stations, the difference in skills for the interpolation methods may appear, even for IDW (k=3) and DWA (k=4), where the only difference between these two methods is the number of neighbouring points used for the interpolation. Upon comparison of the errors and skills at a grid point within the catchment, DWA was found to be better than other interpolation methods, also reported in other studies [4], [28], [29]. Note, Chen and Liu [28] and Hsieh et al.[29] used rainfall data from the rain gauge

stations, whereas Yang et al. [4] used generated data, but all reported that IDW performs better.

For the sake of brevity, when comparing temporal skill over the catchment, only RMSE, CC, ACC and IA are considered. For the spatial comparison, a specific threshold for skill values is set, and the number of grids covered for the thresholds are counted. The spatial comparison reveals that the conservative methods performed much better than the other five interpolation methods, with SOC outperforming FOC. It appears that maintaining the spatial distribution of the precipitation by interpolating in a conservative manner is the main reason behind the better spatial skills of these conservative methods. In conservative methods, the precipitation flux is conserved when interpolated from the source grid onto an interpolated grid. The conservation of flux while interpolating spatially is important, especially for the discontinuous variable like precipitation and due to its high temporal and spatial variability. For instance, if few grid points have no precipitation while others have large values, then bilinear

interpolation can make all points zero, including the large values as it uses four grid points nearest the 2-degree target grid point. In this case, conservative interpolation would be a good approach. During the spatial interpolation, it is presumed that an accurate approximation of the flux on a source grid leads to a more accurate remapping, as evidenced by the used of SOC. In the SOC method, the area-weighted distance from the source cell centroid is considered as the gradient of flux for the interpolated cell [30]. Jones [30] compared first and second-order conservative with the other different interpolation methods and found that conservative methods perform much better for the dataset on regular rectangular grid, where second-order conservative shows an order-of-magnitude improvement over the first order.

Wagner et al. [20] suggested that the spatial skills of the interpolation methods must be considered rather than the skill measured at points. Maintaining the spatial distribution is more important to assess climate variability at a local scale, especially for the precipitation. said the results revealed that the conservative methods would suit better for spatial interpolation of precipitation as they maintain the spatial distribution of the interpolated variables by conserving the flux. Furthermore, SOC may be the best option for the spatial interpolating the gridded precipitation dataset like those from the GCMs as found in this study. This finding is inline with the previous study[30], where the second-order conservative (SOC) method was found to be an appropriate choice for interpolating the gridded dataset. For the cross-validation, similar studies at other catchments/regions are recommended.

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